



WODCON XXI

One Or More Solutions
For Cutting Forces
In Clay & Rock

Dr.ir. Sape A. Miedema
Head of Studies

**MSc Offshore & Dredging Engineering
& Marine Technology
&**

**Associate Professor of
Dredging Engineering**

Delft University of Technology



Goals & Targets

Problem Definition

In rock cutting the rock will fail either by brittle shear failure or by brittle tensile failure. How do these failure mechanisms depend on the UCS and the BTS values of the rock and how is the transition between those two mechanisms.

Solution

At high UCS/BTS ratios there is brittle tensile failure, at low ratios there is brittle shear failure. In between there is an area where both may occur simultaneously, resulting in the chip type of cutting process.





Rock Cutting Introduction



Rock Cutter Heads



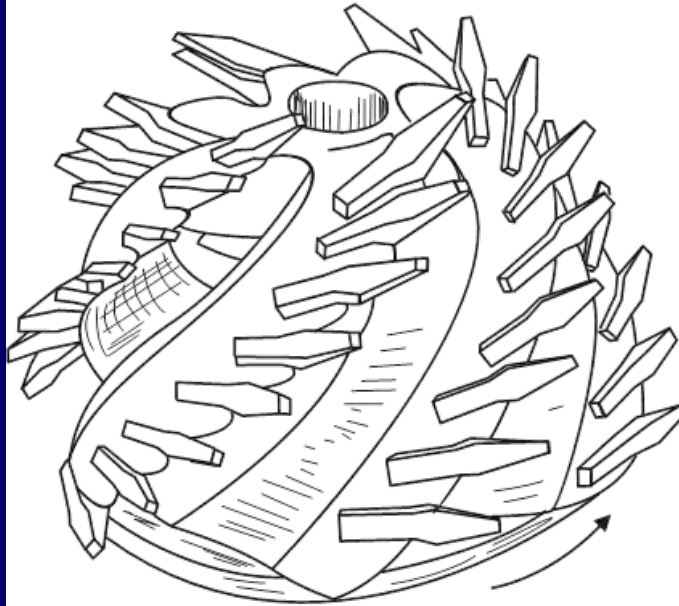
"PICK-POINT"



NARROW CHISELS



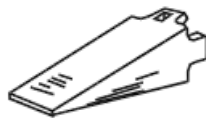
TRAPEZOIDALE PICK-POINT



NORMAL HELIX CUTTER



REVERSE HELIX CUTTER



WIDE CHISEL



CL FLARED



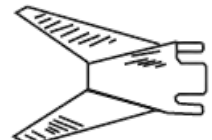
BELOW CL FLARED

TYPE A



BELOW CL FLARED

TYPE B (CLAY FLARE)

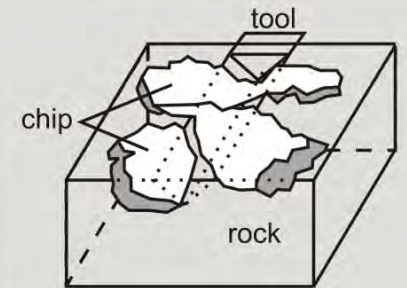
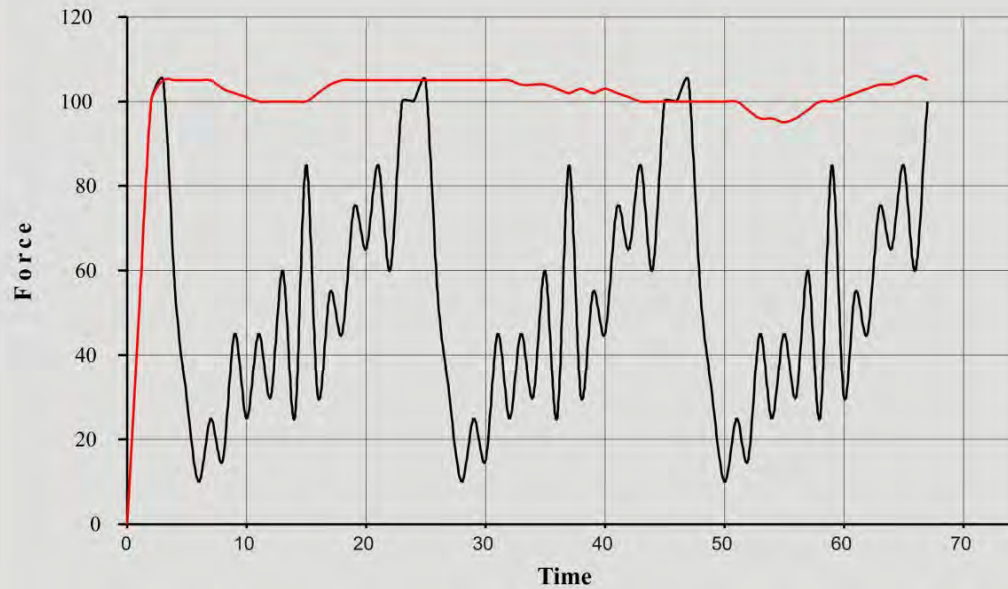


"DEVIL TEETH"
(FLORIDA)

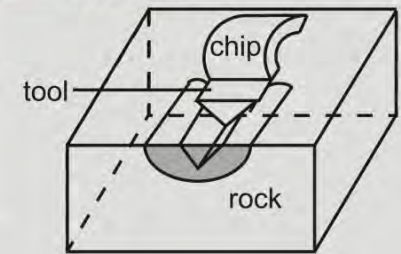
Brittle versus Ductile



Brittle & Ductile Cutting



— Brittle
— Ductile

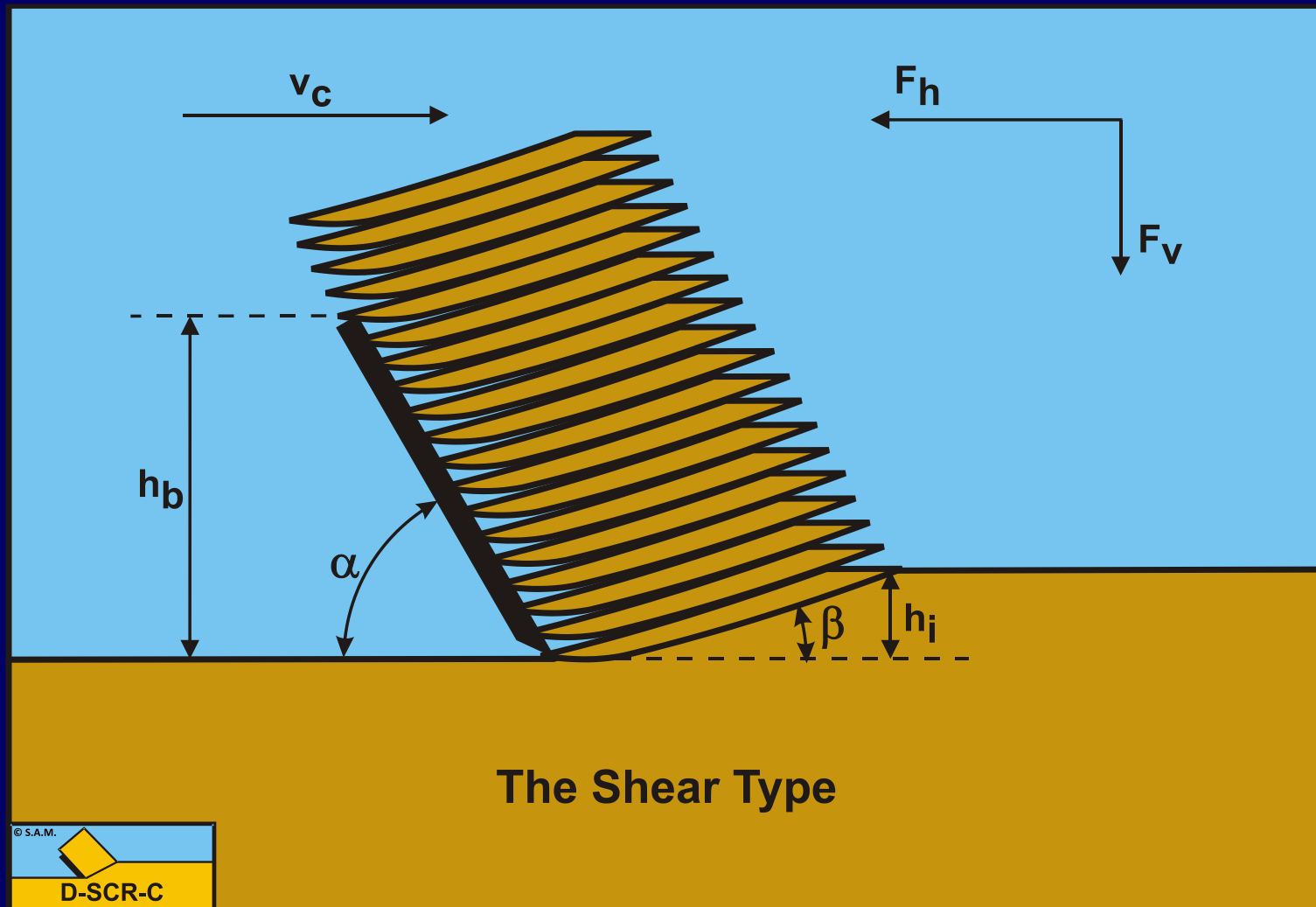




Failure Mechanisms

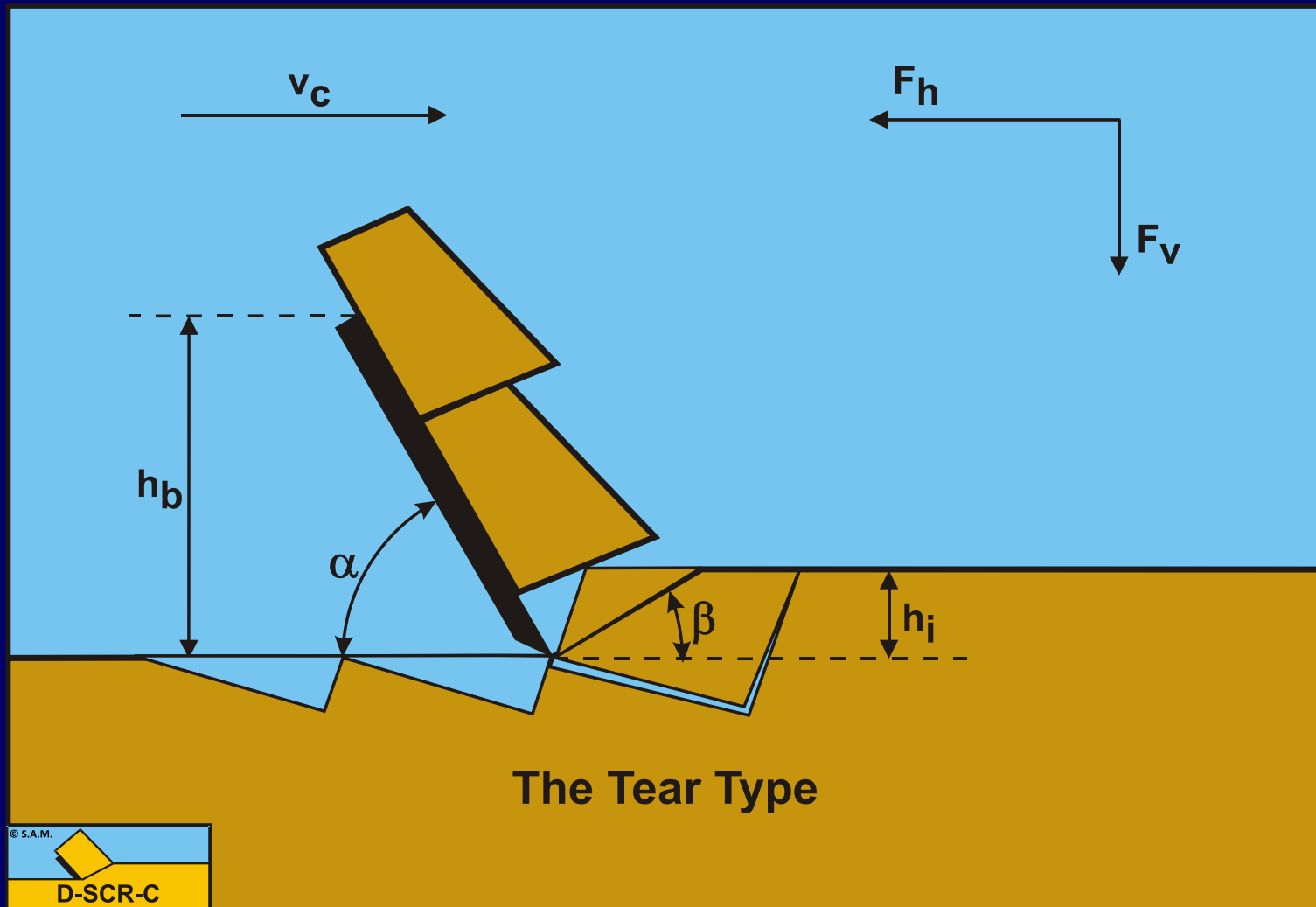


Brittle Shear Failure/Shear Type

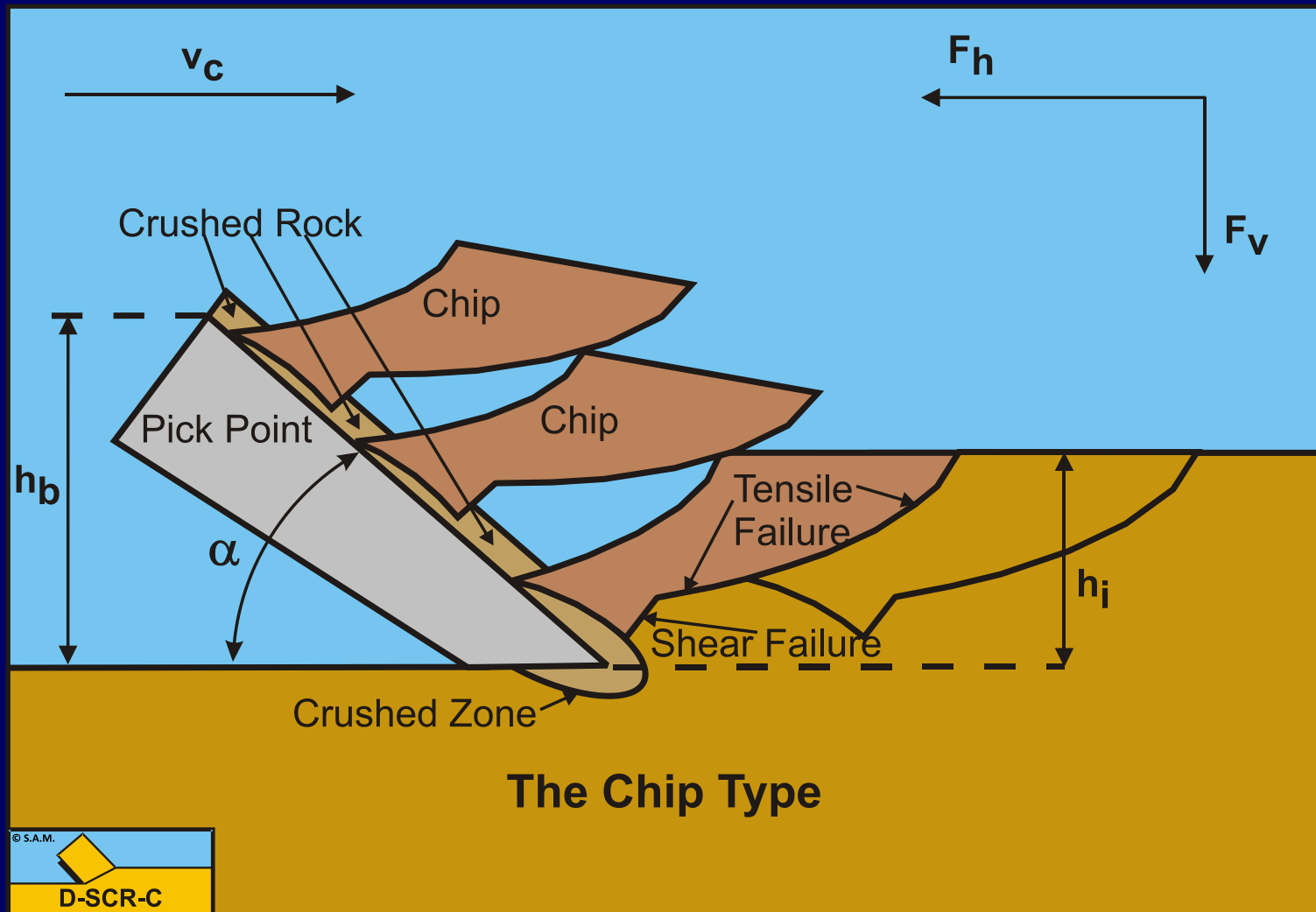




Brittle Tensile Failure/Tear Type



Shear & Tensile Failure/Chip Type

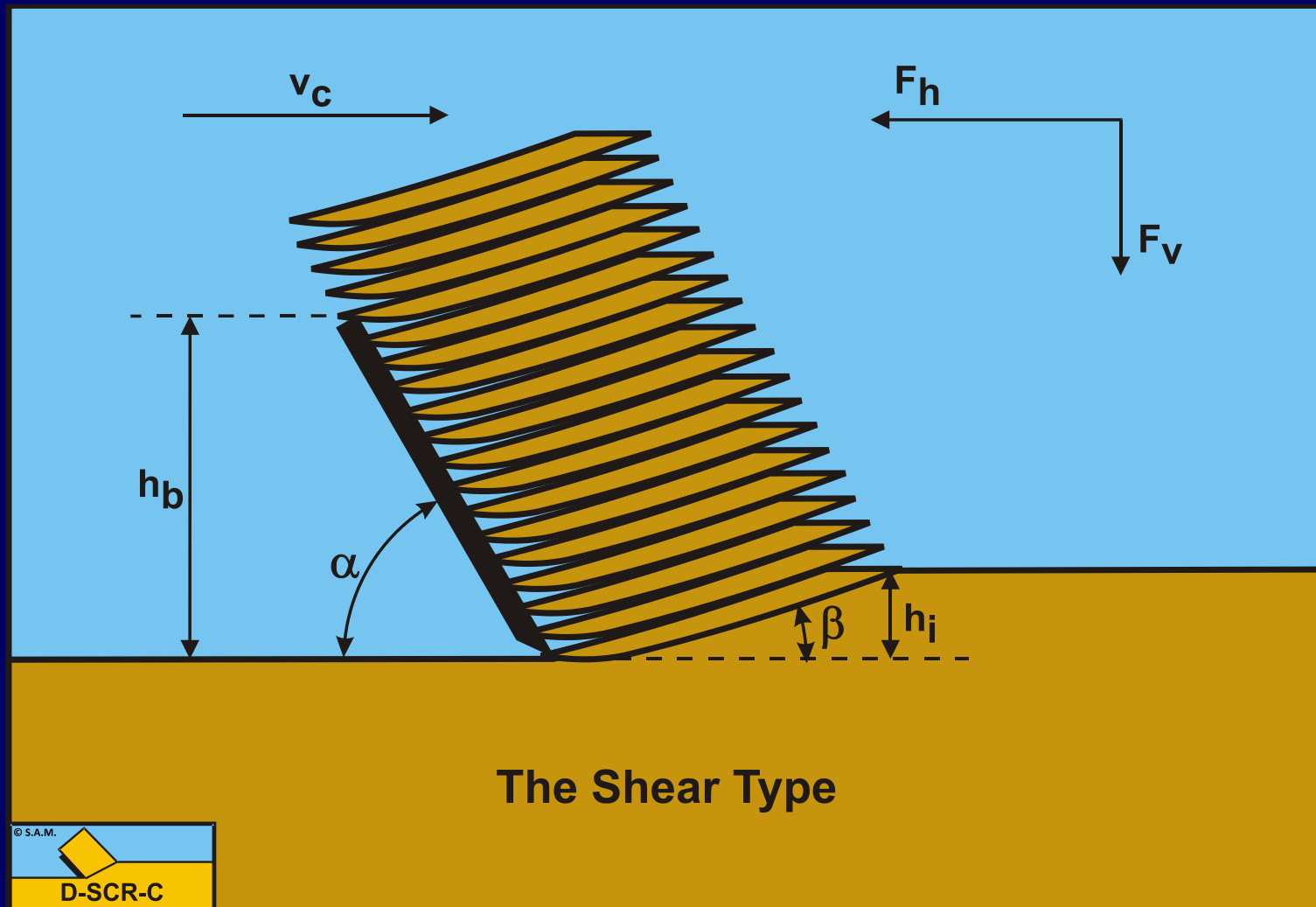




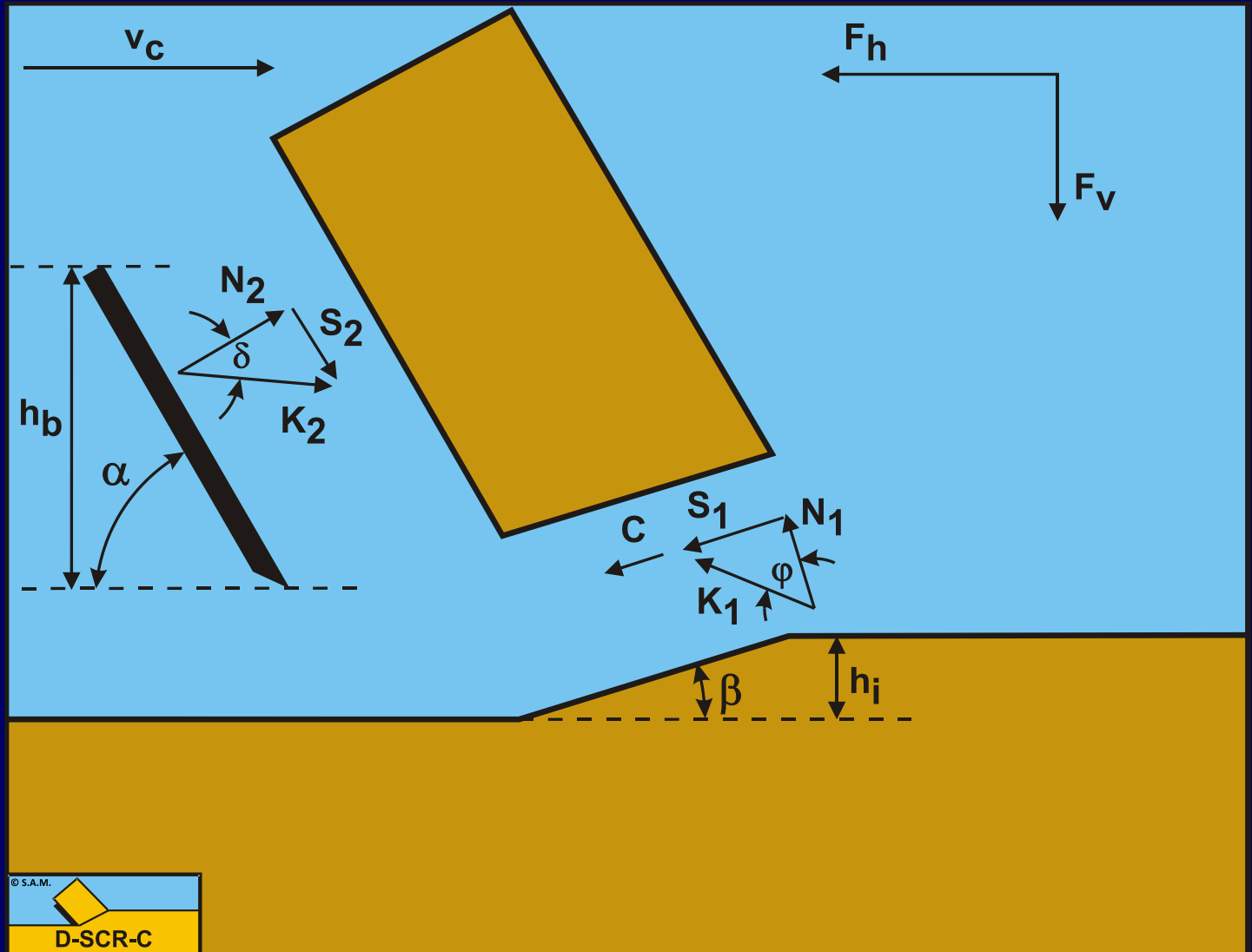
Shear Failure/Shear Type Peak Forces



Brittle Shear Failure/Shear Type



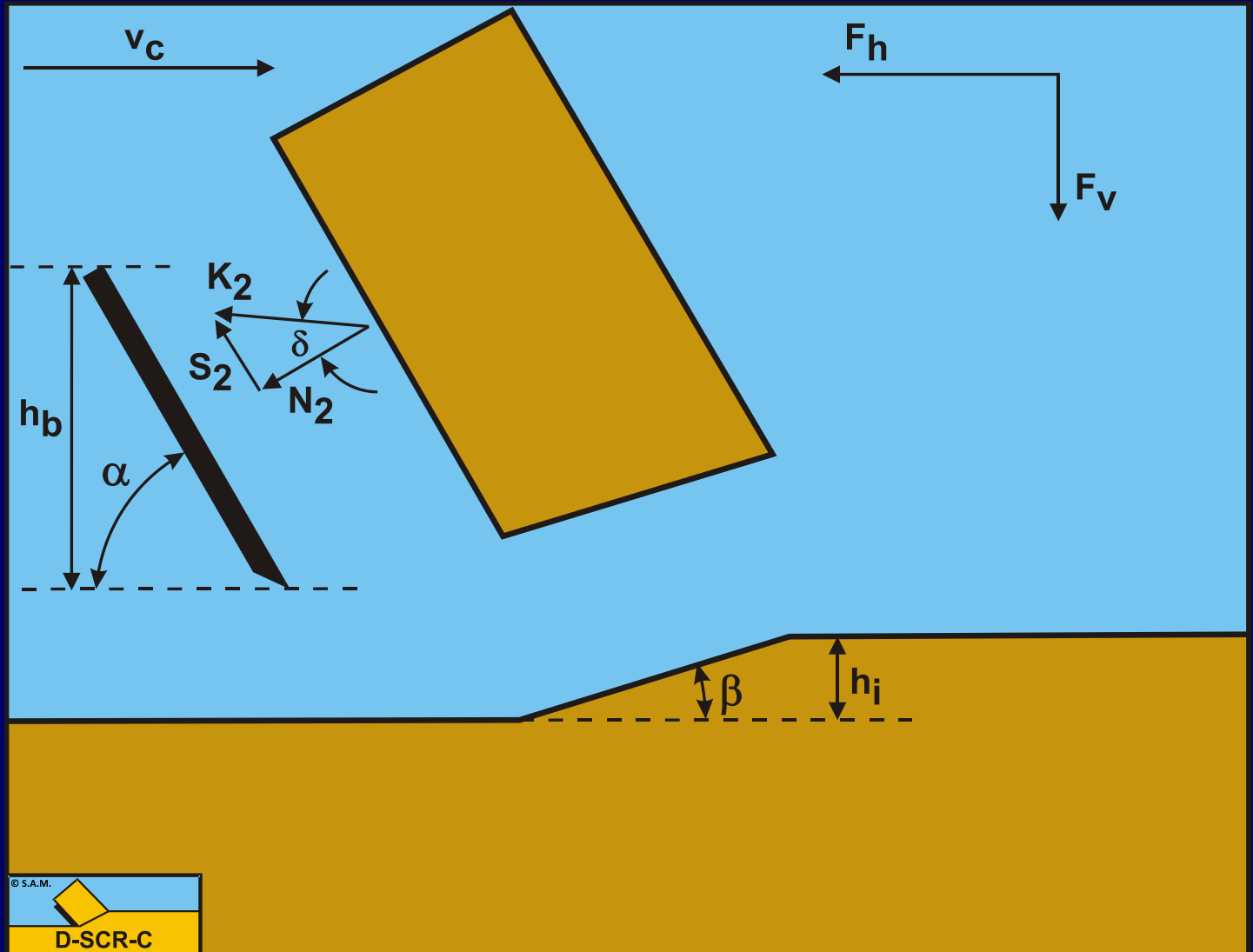
Forces on the Layer Cut



© S.A.M.

D-SCR-C

Forces on the Blade



© S.A.M.

D-SCR-C



Resulting Cutting Forces

$$c = \frac{\text{UCS}}{2} \cdot \left(\frac{1 - \sin(\varphi)}{\cos(\varphi)} \right)$$

$$C = \frac{c \cdot h_i \cdot w}{\sin(\beta)}$$

$$\beta = \frac{\pi}{2} - \frac{\alpha + \delta + \varphi}{2}$$

$$F_h = \frac{2 \cdot c \cdot h_i \cdot w \cdot \cos(\varphi) \cdot \sin(\alpha + \delta)}{1 + \cos(\alpha + \delta + \varphi)} = \lambda_{\text{HF}} \cdot c \cdot h_i \cdot w$$

$$F_v = \frac{2 \cdot c \cdot h_i \cdot w \cdot \cos(\varphi) \cdot \cos(\alpha + \delta)}{1 + \cos(\alpha + \delta + \varphi)} = \lambda_{\text{VF}} \cdot c \cdot h_i \cdot w$$

Stresses on the Shear Plane

$$c = \frac{\text{UCS}}{2} \cdot \left(\frac{1 - \sin(\varphi)}{\cos(\varphi)} \right)$$

$$N_1 = \frac{C \cdot \sin\left(\frac{\alpha + \delta - \varphi}{2}\right)}{\cos\left(\frac{\alpha + \delta + \varphi}{2}\right)} \cdot \cos(\varphi)$$

$$\sigma_1 = \frac{c \cdot \sin\left(\frac{\alpha + \delta - \varphi}{2}\right)}{\cos\left(\frac{\alpha + \delta + \varphi}{2}\right)} \cdot \cos(\varphi)$$

$$\tau_1 = c + \sigma_1 \cdot \tan(\varphi)$$





Failure Criteria The Mohr Circle



Failure Criteria

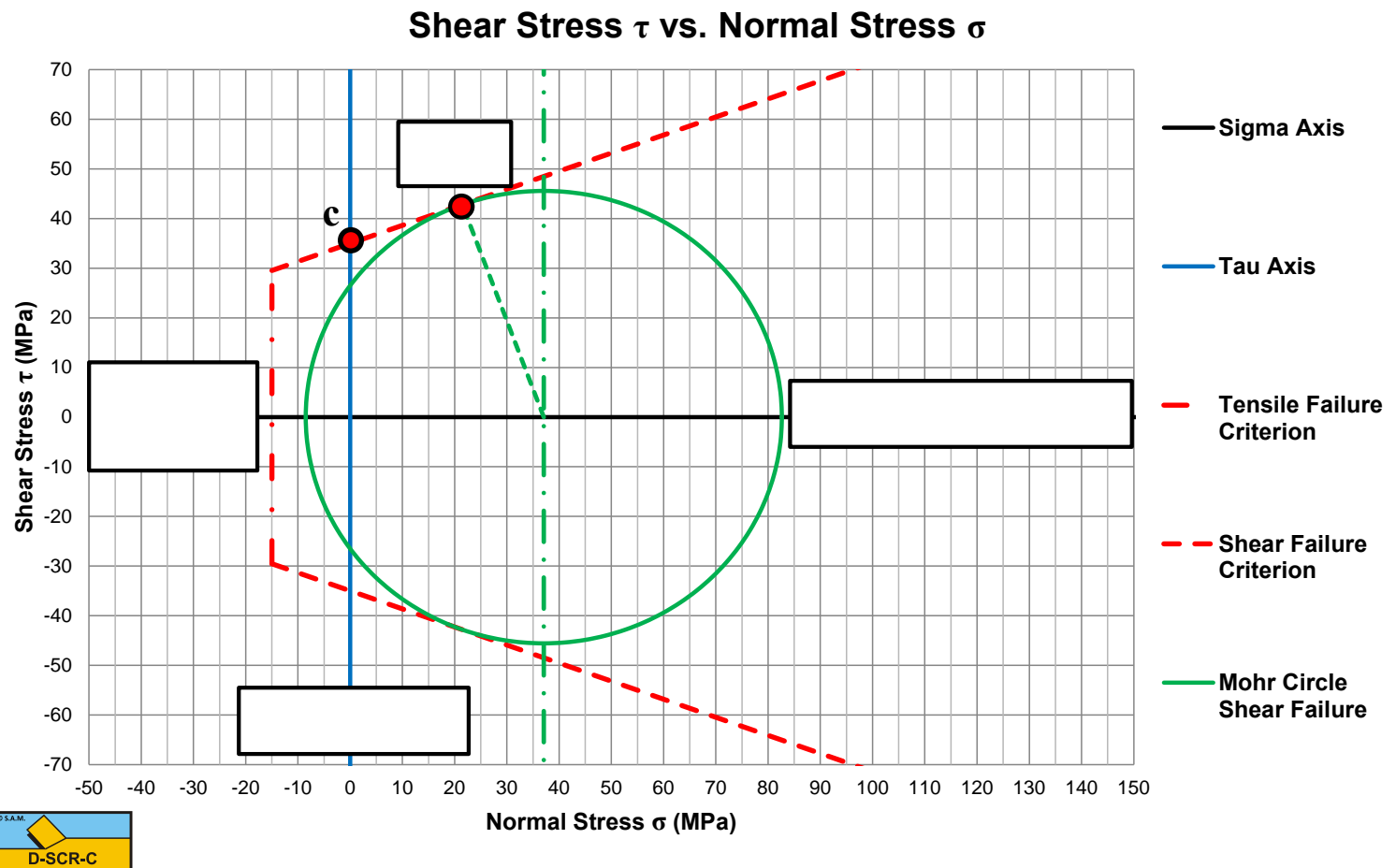
The Mohr circle can never cross or touch one of the failure criteria. These criteria are:

- The shear failure criterion.
- The tensile failure criterion.

If the Mohr circle touches or crosses one of the failure criteria, failure will occur.



The Mohr Circle, $BTS = -15 \text{ MPa}$



$UCS = 100 \text{ MPa}$, $BTS = -15 \text{ MPa}$, $\alpha = 60^\circ$, $\phi = 20^\circ$,
 $h_i = 0.1 \text{ m}$ & $w = 0.1 \text{ m}$

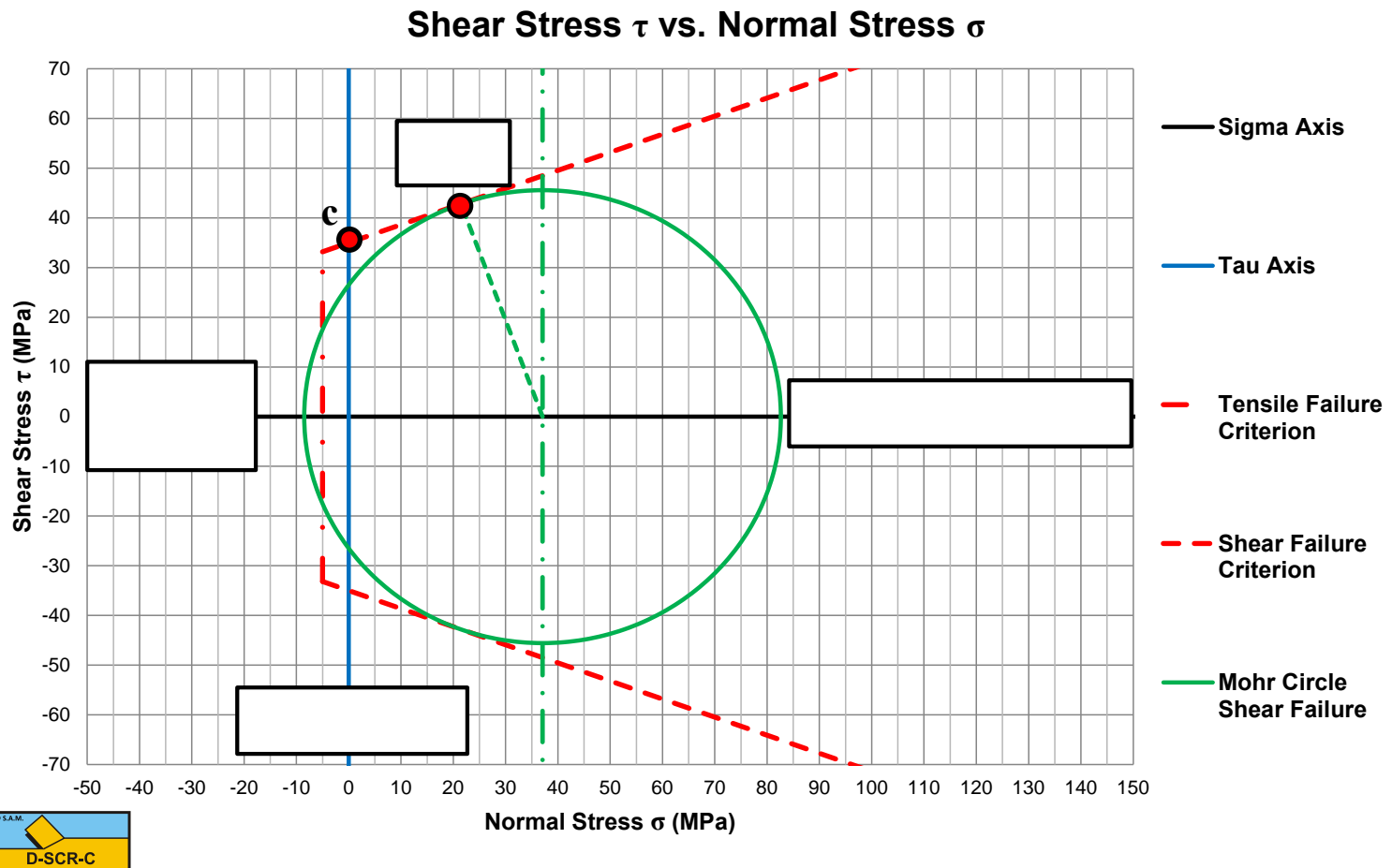
Delft University of Technology – Offshore & Dredging Engineering



Tensile Failure/Tear Type Peak Forces



The Mohr Circle, $BTS = -5 \text{ MPa}$

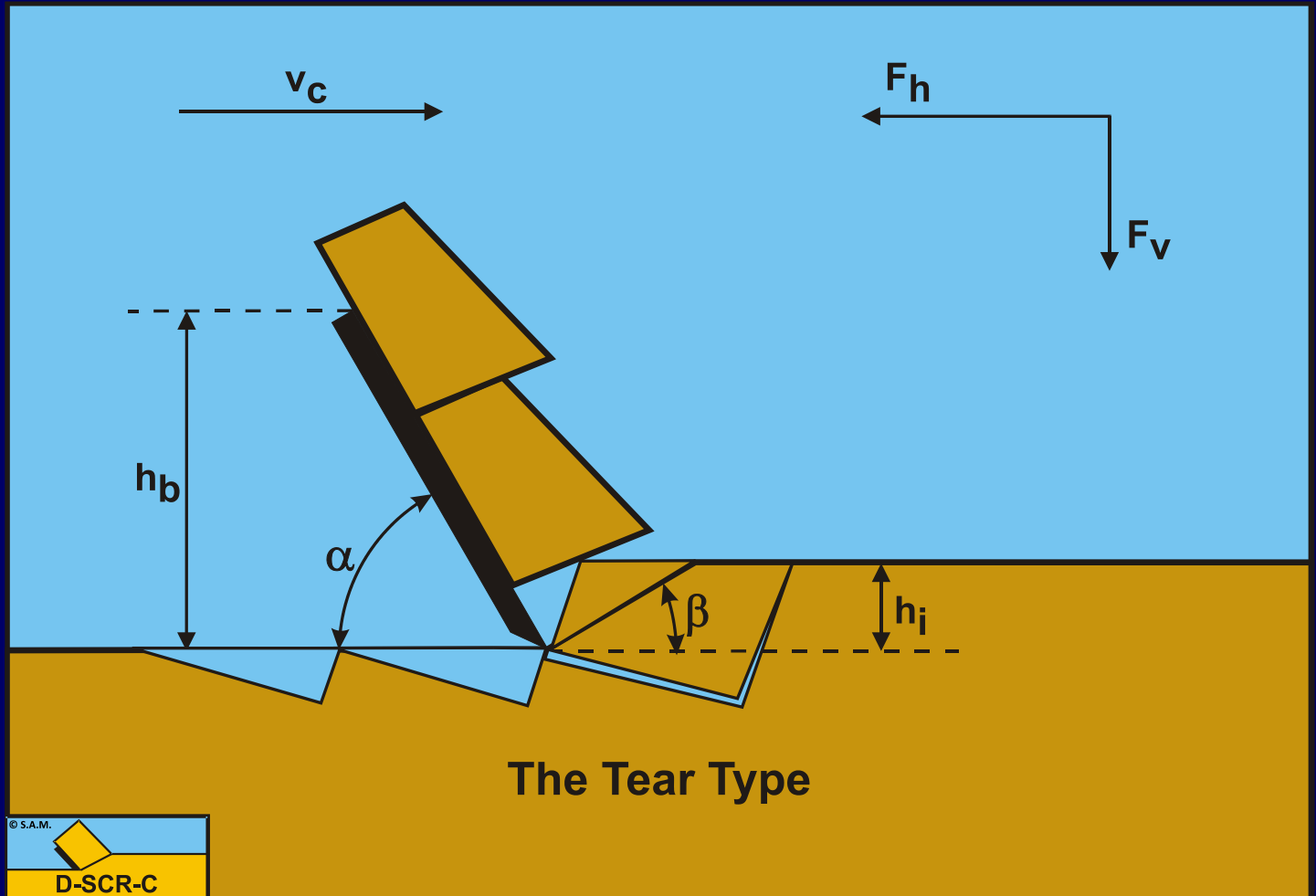


$UCS = 100 \text{ MPa}$, $BTS = -5 \text{ MPa}$, $\alpha = 60^\circ$, $\varphi = 20^\circ$,
 $h_i = 0.1 \text{ m}$ & $w = 0.1 \text{ m}$

Delft University of Technology – Offshore & Dredging Engineering



Brittle Tensile Failure/Tear Type





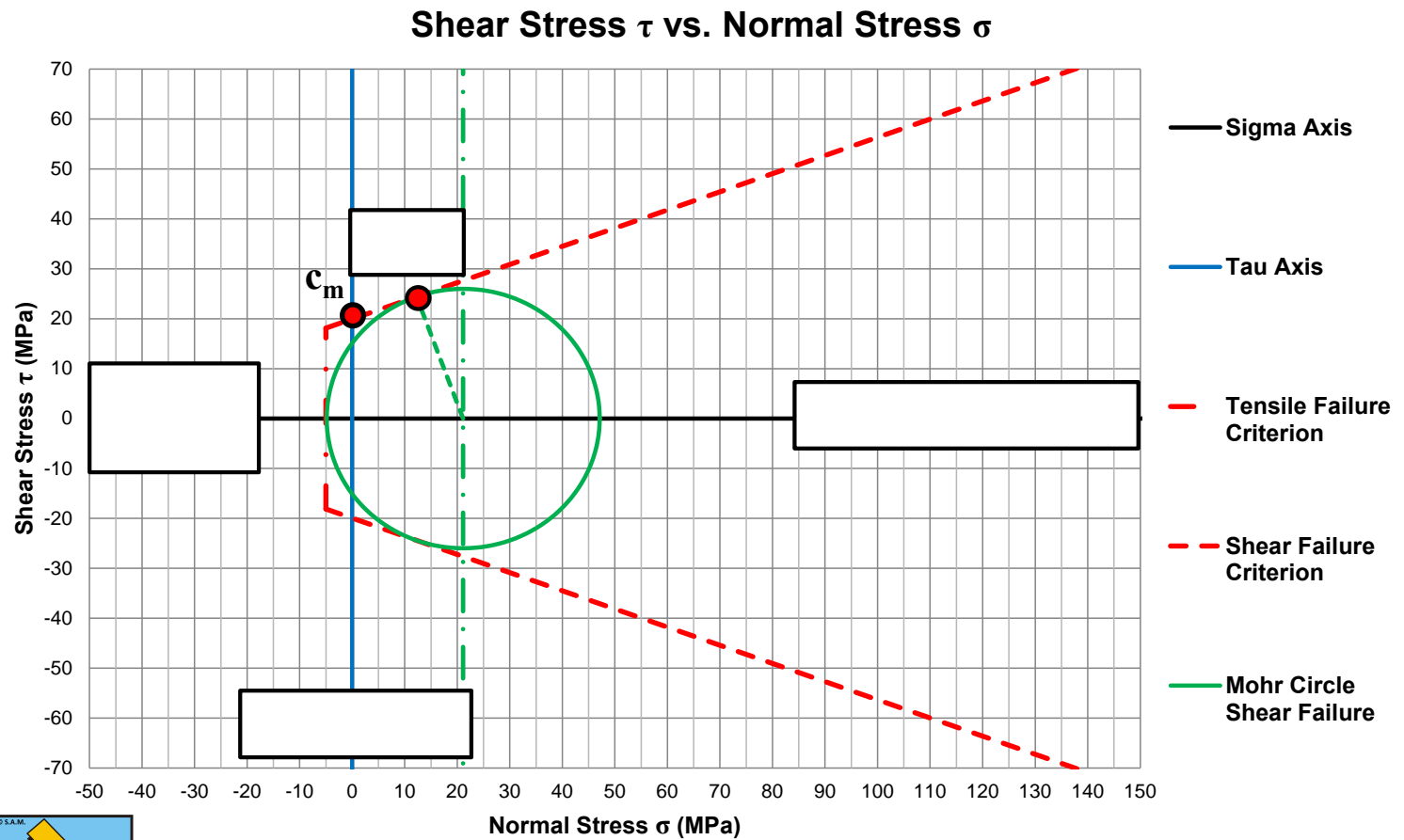
The Reduced Mohr Circle

The Mohr circle can never cross or touch one of the failure criteria. So if it crosses the tensile strength criterion, tensile failure will occur.

This can be modeled by defining a mobilized shear strength (cohesion) that creates a Mohr circle just touching the tensile strength criterion, based on the tensile strength.

This mobilized cohesion however also results in a different shear angle β .

The Reduced Mohr Circle, $BTS = -5$ MPa



$UCS = 57$ MPa, $BTS = -5$ MPa, $\alpha = 60^\circ$, $\varphi = 20^\circ$,
 $h_i = 0.1$ m & $w = 0.1$ m

Delft University of Technology – Offshore & Dredging Engineering



Mobilized Shear Strength

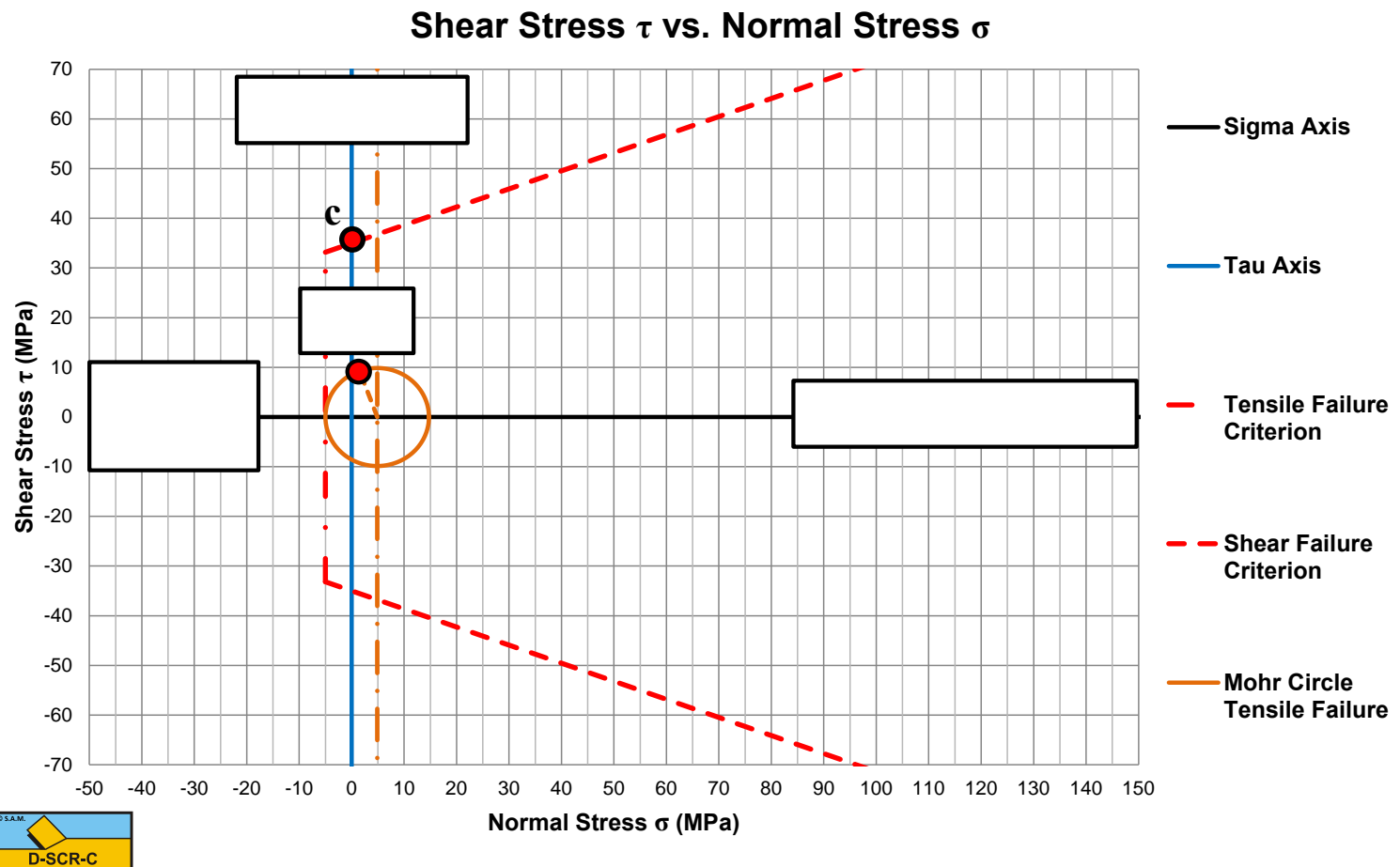
$$F_h = \frac{2 \cdot c_m \cdot h_i \cdot w \cdot \cos(\varphi) \cdot \sin(\alpha + \delta)}{\cos(\pi/4) + \cos(\alpha + \delta + \varphi)} = \lambda_{HT} \cdot \sigma_T \cdot h_i \cdot w$$

$$F_v = \frac{2 \cdot c_m \cdot h_i \cdot w \cdot \cos(\varphi) \cdot \cos(\alpha + \delta)}{\cos(\pi/4) + \cos(\alpha + \delta + \varphi)} = \lambda_{VT} \cdot \sigma_T \cdot h_i \cdot w$$

$$c_m = \frac{\sigma_T}{\left(\frac{\sin\left(\frac{\alpha + \delta - \varphi - \pi/4}{2}\right)}{\cos\left(\frac{\alpha + \delta + \varphi - \pi/4}{2}\right)} - 1 \right) \cdot \left(\frac{1 - \sin(\varphi)}{\cos(\varphi)} \right)}$$

$$\beta = \frac{\pi}{2} - \frac{\pi/4 + \alpha + \delta + \varphi}{2}$$

The Reduced Mohr Circle, BTS=-5 MPa



$UCS=100 \text{ MPa}$, $BTS=-5 \text{ MPa}$, $\alpha=60^\circ$, $\varphi=20^\circ$,
 $h_i=0.1 \text{ m}$ & $w=0.1 \text{ m}$

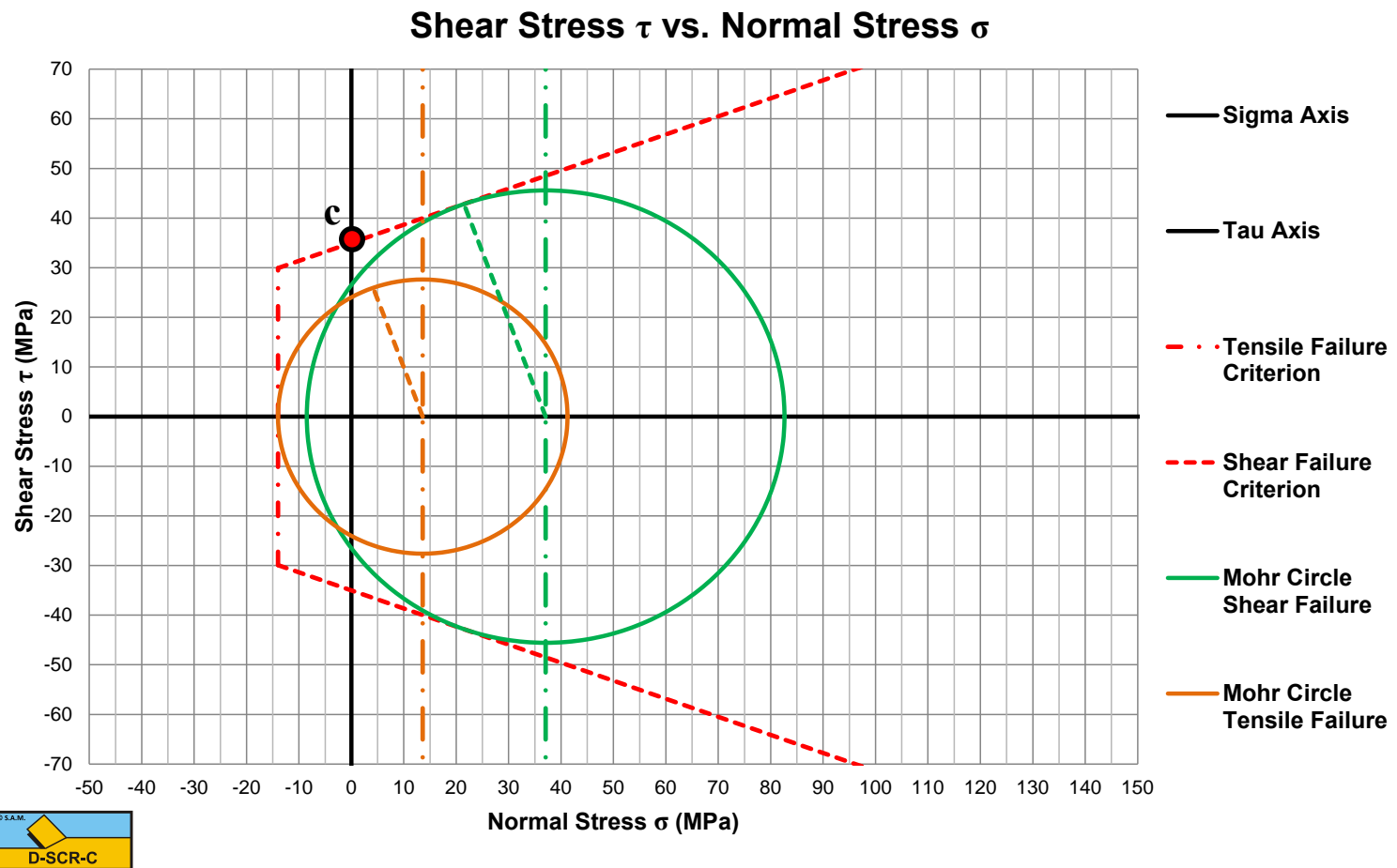
Delft University of Technology – Offshore & Dredging Engineering



Two Solutions?



The Mohr Circles, $BTS = -14$ MPa



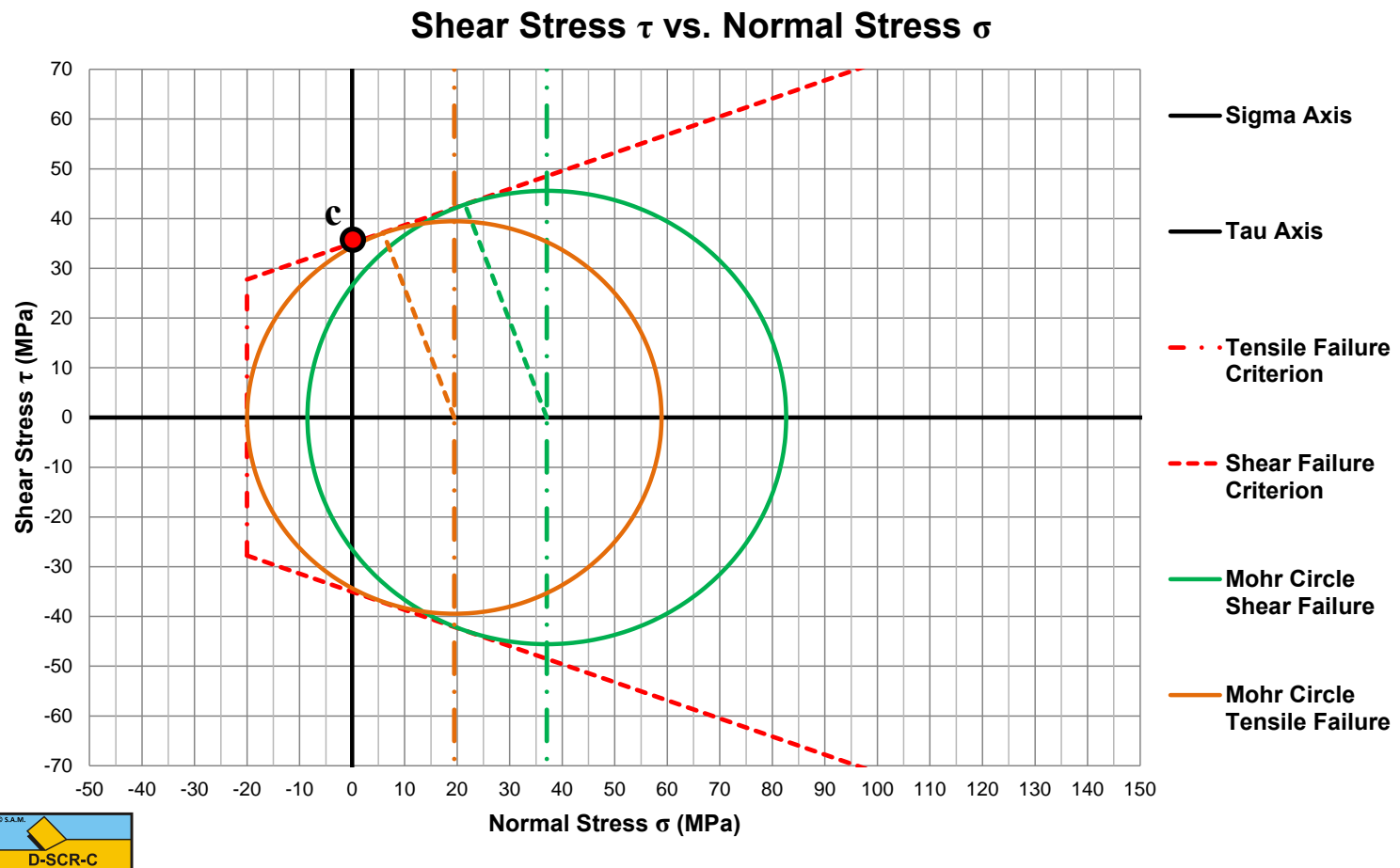
$UCS = 100$ MPa, $BTS = -14$ MPa, $\alpha = 60^\circ$, $\varphi = 20^\circ$,
 $h_i = 0.1$ m & $w = 0.1$ m

Delft University of Technology – Offshore & Dredging Engineering



Two Limits The Chip Type

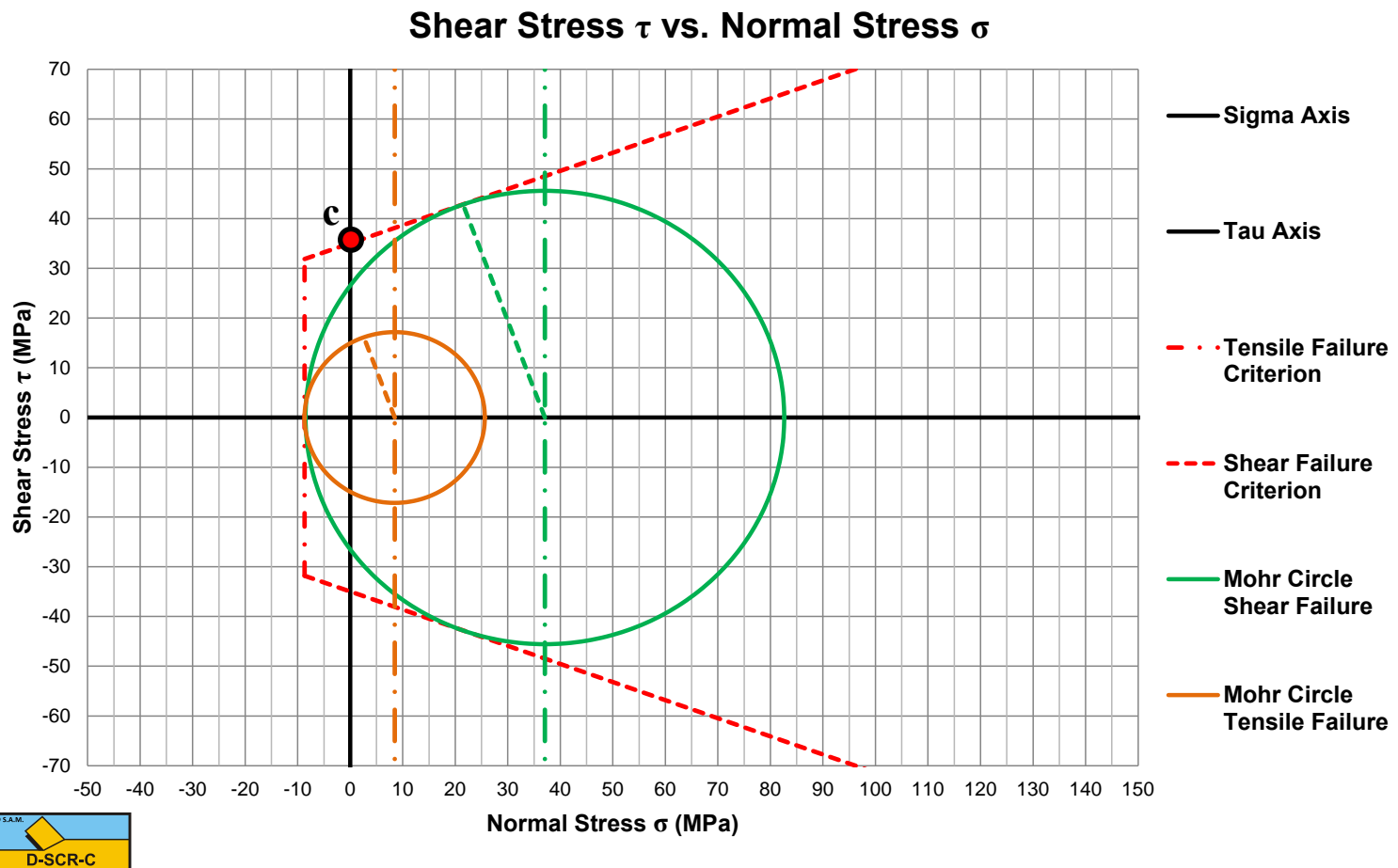
The Mohr Circles, Lower Limit



$UCS=100 \text{ MPa}$, $BTS=-20 \text{ MPa}$, $\alpha=60^\circ$, $\varphi=20^\circ$,
 $h_i=0.1 \text{ m}$ & $w=0.1 \text{ m}$

Delft University of Technology – Offshore & Dredging Engineering

The Mohr Circles, Upper Limit



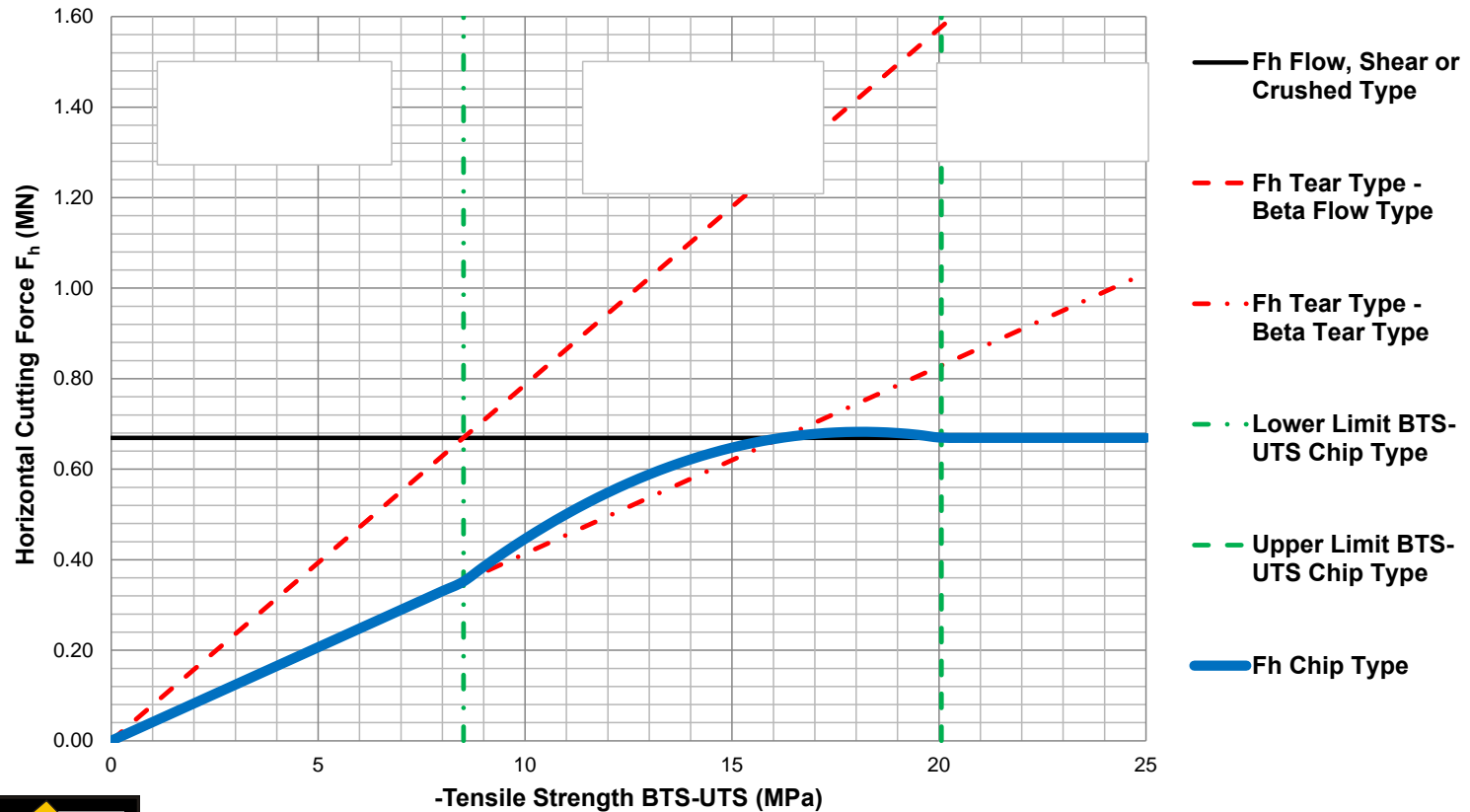
$UCS=100 \text{ MPa}$, $BTS=-8.7 \text{ MPa}$, $\alpha=60^\circ$, $\varphi=20^\circ$,
 $h_i=0.1 \text{ m}$ & $w=0.1 \text{ m}$

Delft University of Technology – Offshore & Dredging Engineering



The Horizontal Cutting Force

Horizontal Cutting Force F_h vs. Tensile Strength BTS-UTS



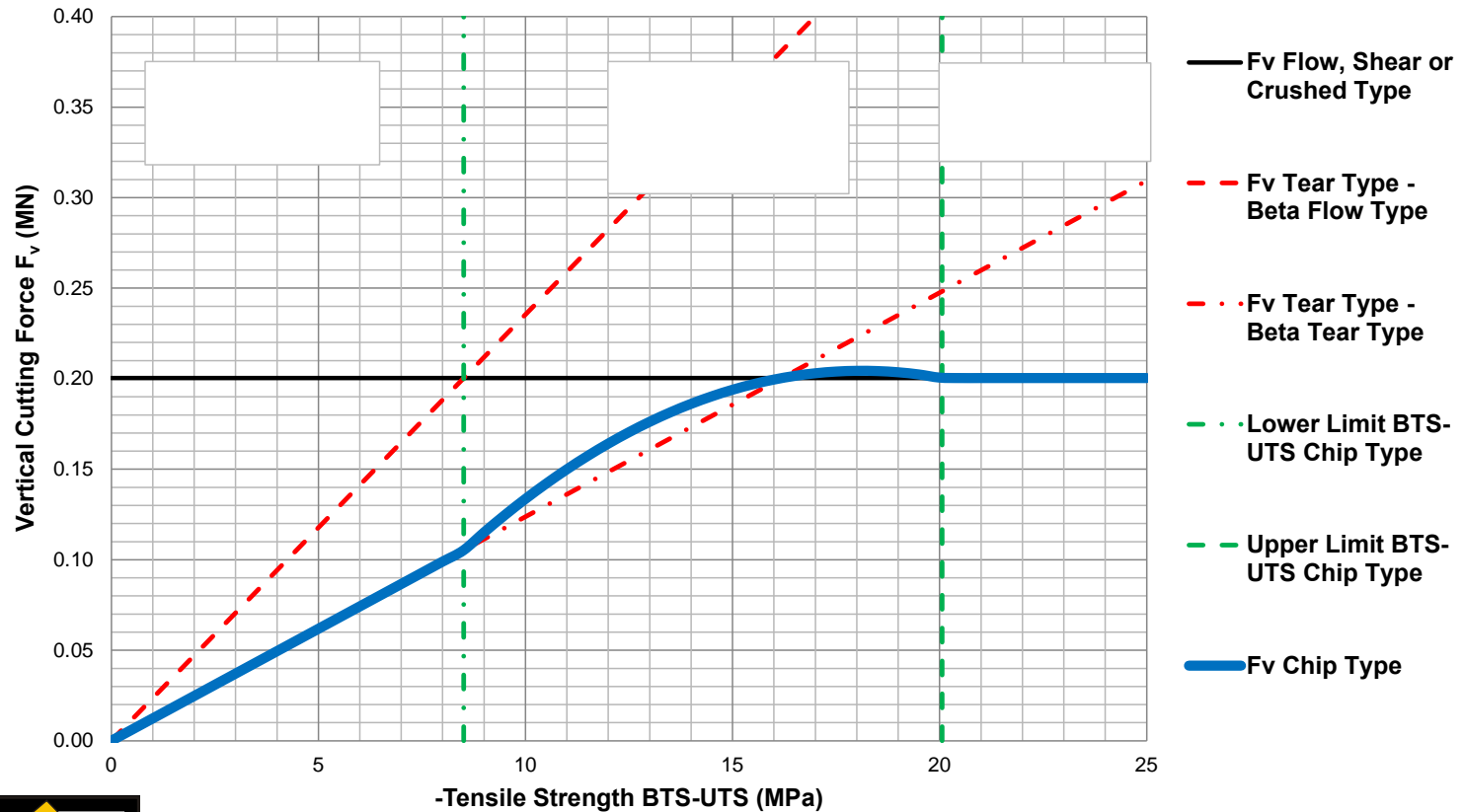
$$UCS=100 \text{ MPa}, \alpha=60^\circ, \varphi=20^\circ,$$

$$h_i=0.1 \text{ m} \ \& \ w=0.1 \text{ m}$$



The Vertical Cutting Force

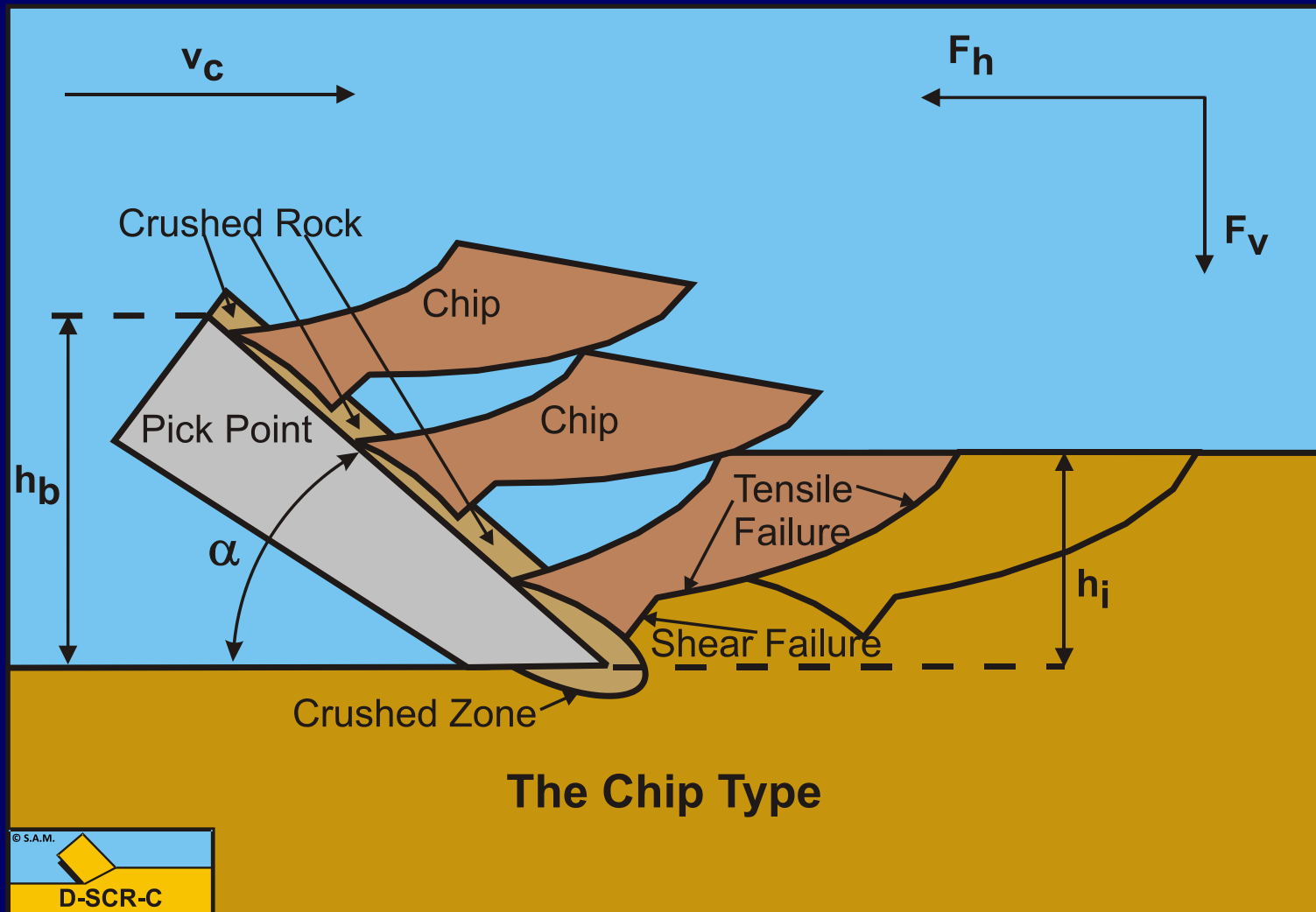
Vertical Cutting Force F_v vs. Tensile Strength BTS-UTS



$$UCS=100 \text{ MPa}, \alpha=60^\circ, \varphi=20^\circ,$$

$$h_i=0.1 \text{ m} \ \& \ w=0.1 \text{ m}$$

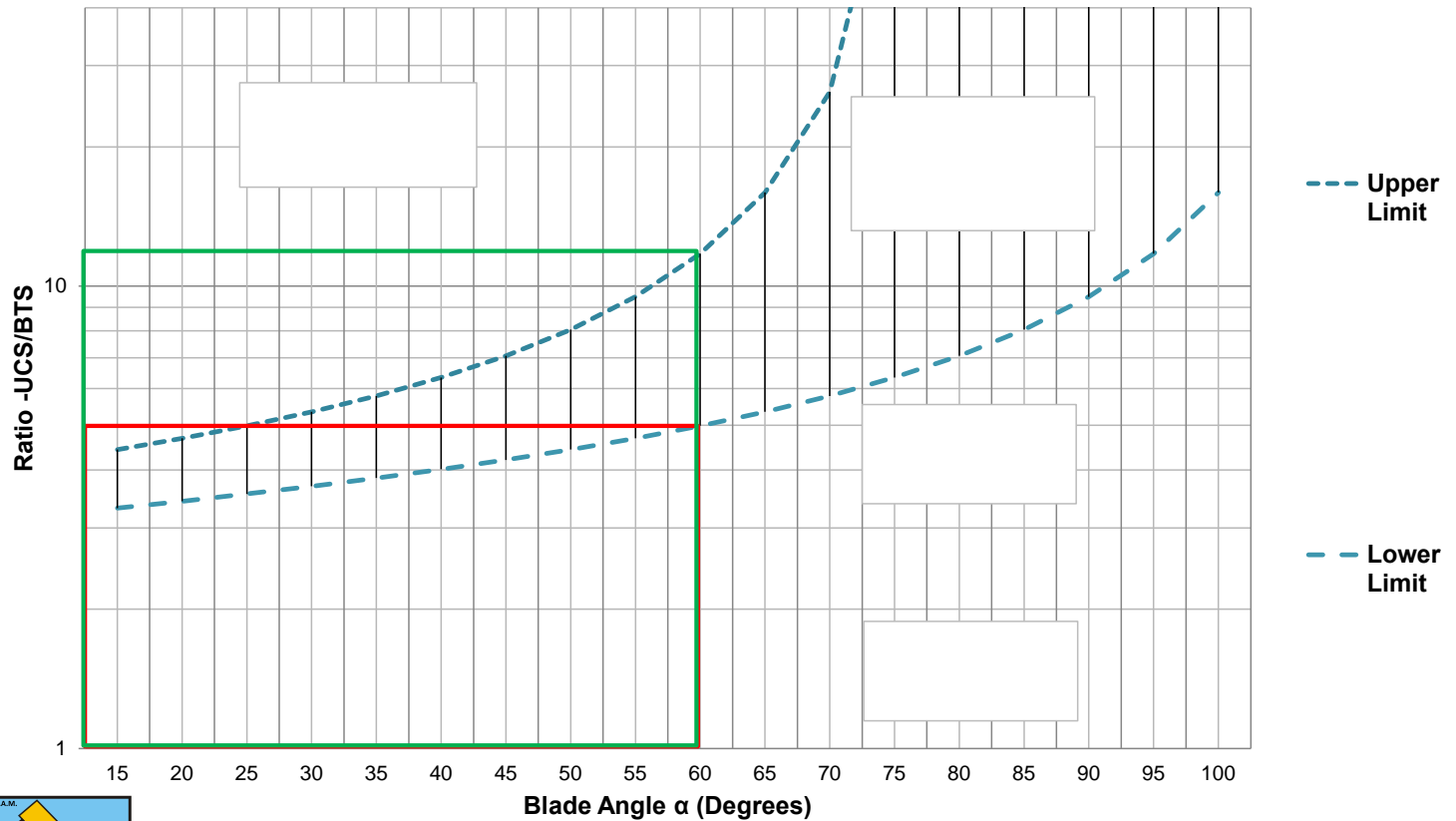
Shear & Tensile Failure/Chip Type





Lower & Upper Limits

A & B: Tensile Failure vs. Shear Failure, $\phi=20^\circ$

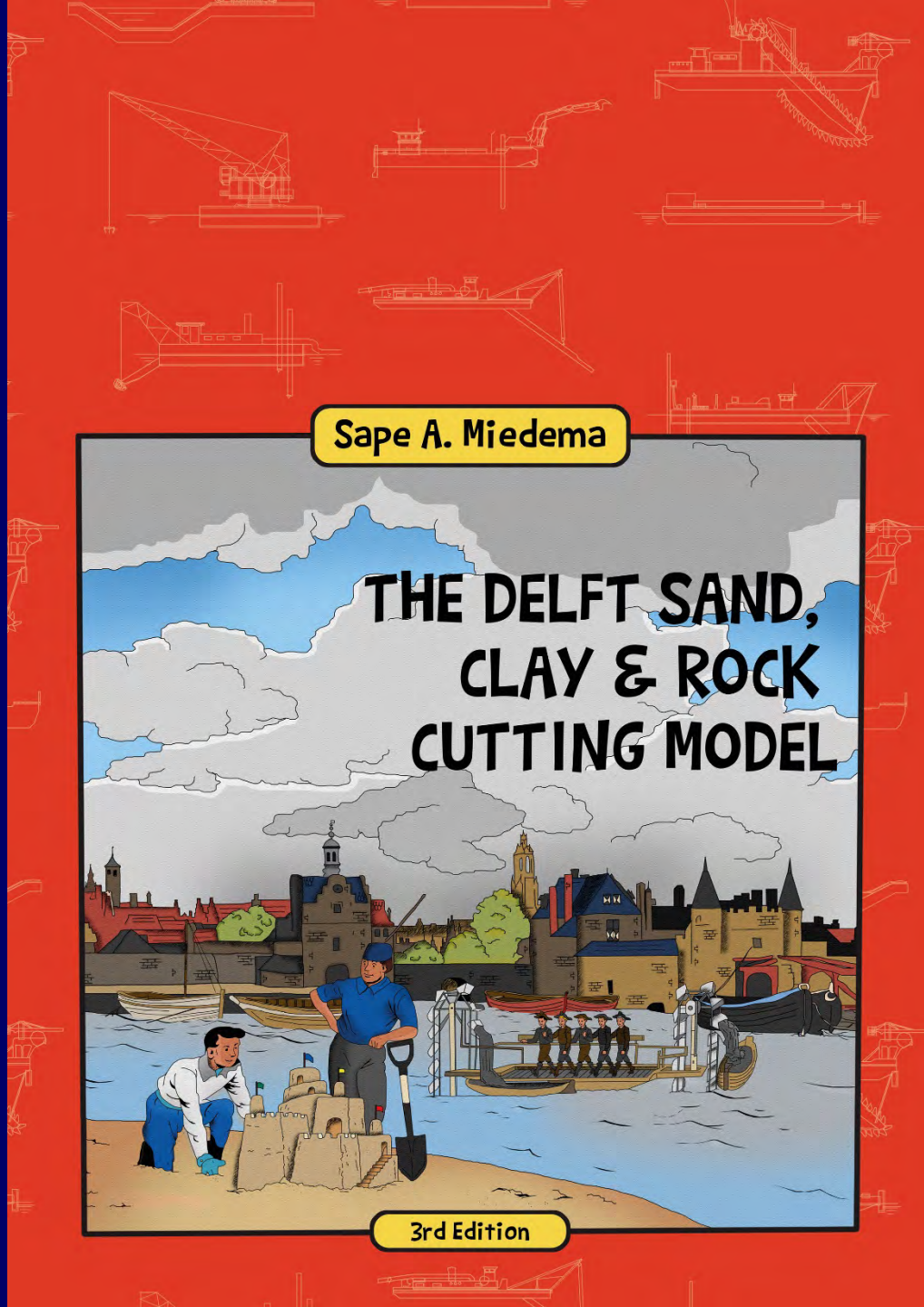


UCS=100 MPa, BTS=-20 MPa, $\alpha=60^\circ$, $\phi=20^\circ$

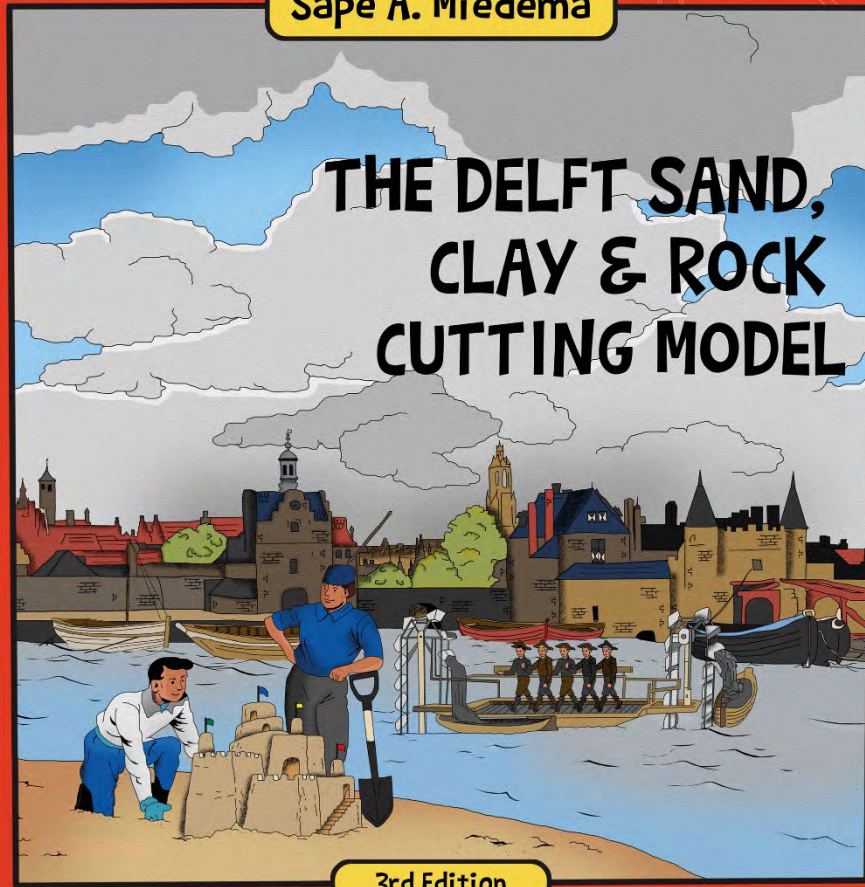


Conclusions

- Below the UCS/BTS lower limit there is always shear failure.
- Above the UCS/BTS upper limit there is always tensile failure.
- Between the two limits there is most probably a combination of both failure mechanisms as is present in the Chip Type.



Sape A. Miedema



THE DELFT SAND, CLAY & ROCK CUTTING MODEL

3rd Edition



Questions?