



# WODCON XXI

## A Comparison Of Different Slurry Transport Models For Sands & Gravels

**Sape A. Miedema  
&  
Robert C. Ramsdell**



# Goals & Targets

## Problem Definition

There are many equations for determining **Head Losses** and the **Limit Deposit Velocity** in slurry transport. How to compare these models and how to determine which models can be applied.

## Solution

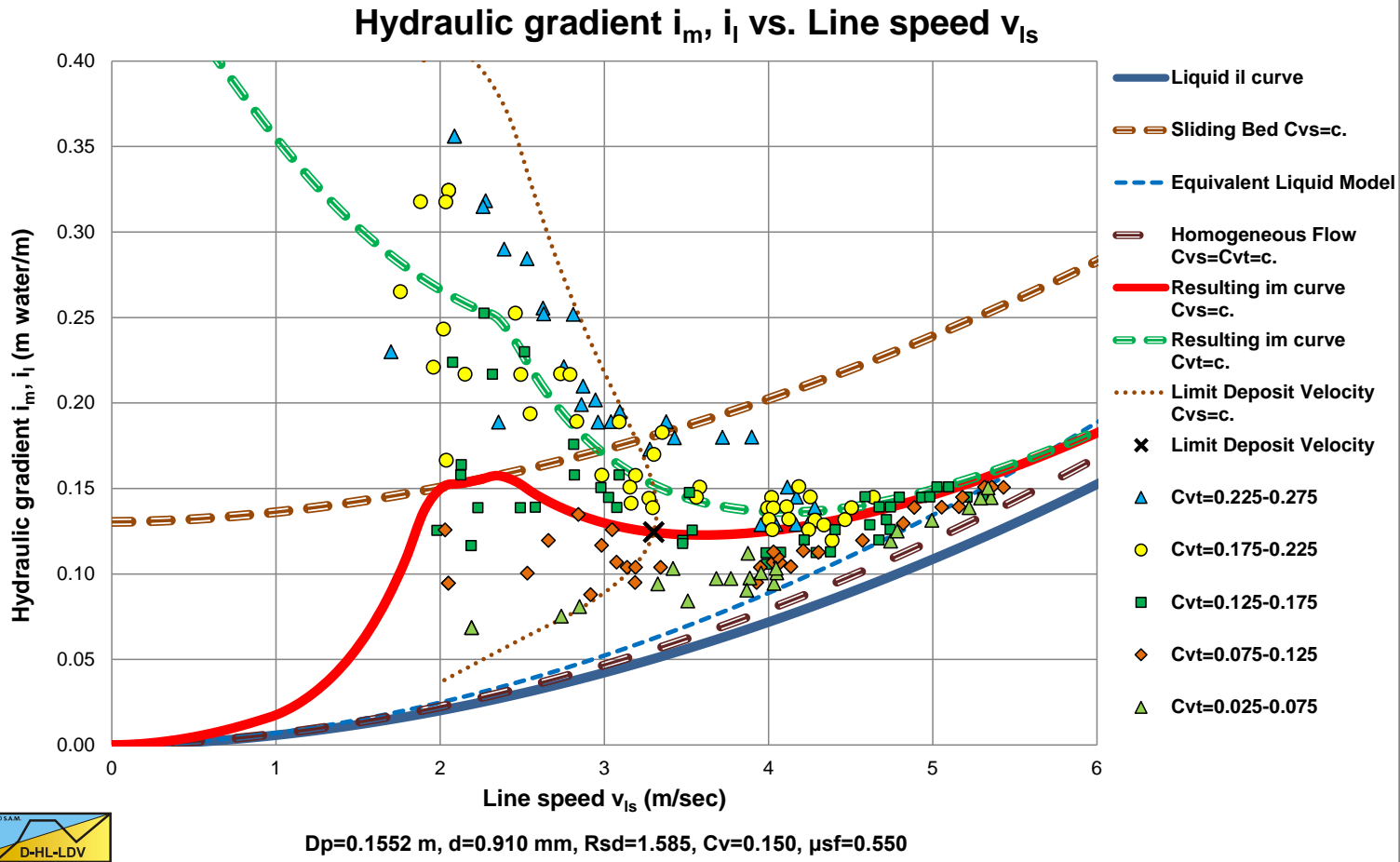
Using the transition line speed of the heterogeneous flow regime with the homogeneous flow regime is a good indicator for the **Head Losses** at operational line speeds. Using the Durand Froude number is a good indicator for the **Limit Deposit Velocity**.





## Introduction

## Data from Yagi et al., $i_m - v_{ls}$

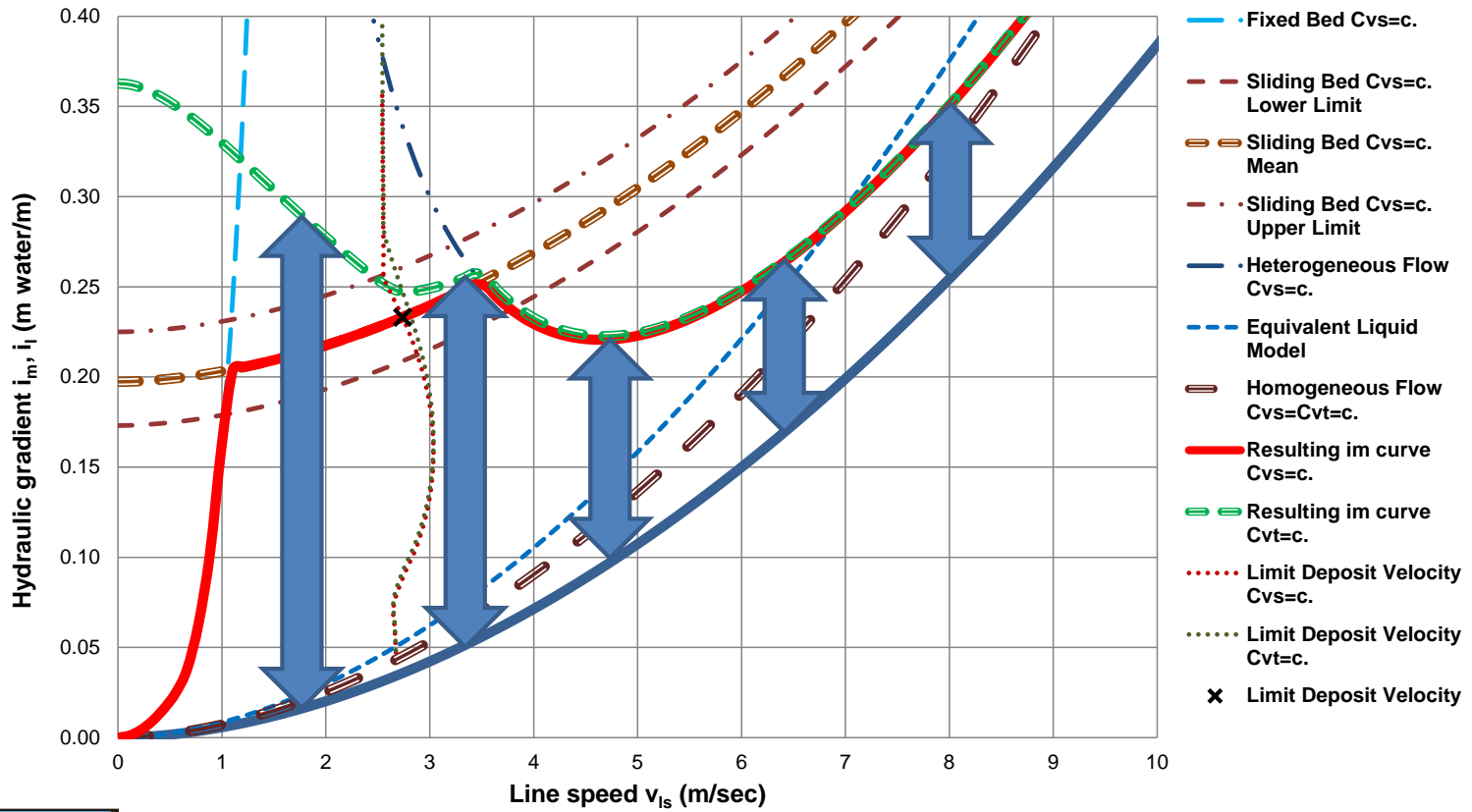


Data looks unorganized depending on the volumetric concentration of the solids.

## Solids Effect



Hydraulic gradient  $i_m, i_l$  vs. Line speed  $v_{ls}$

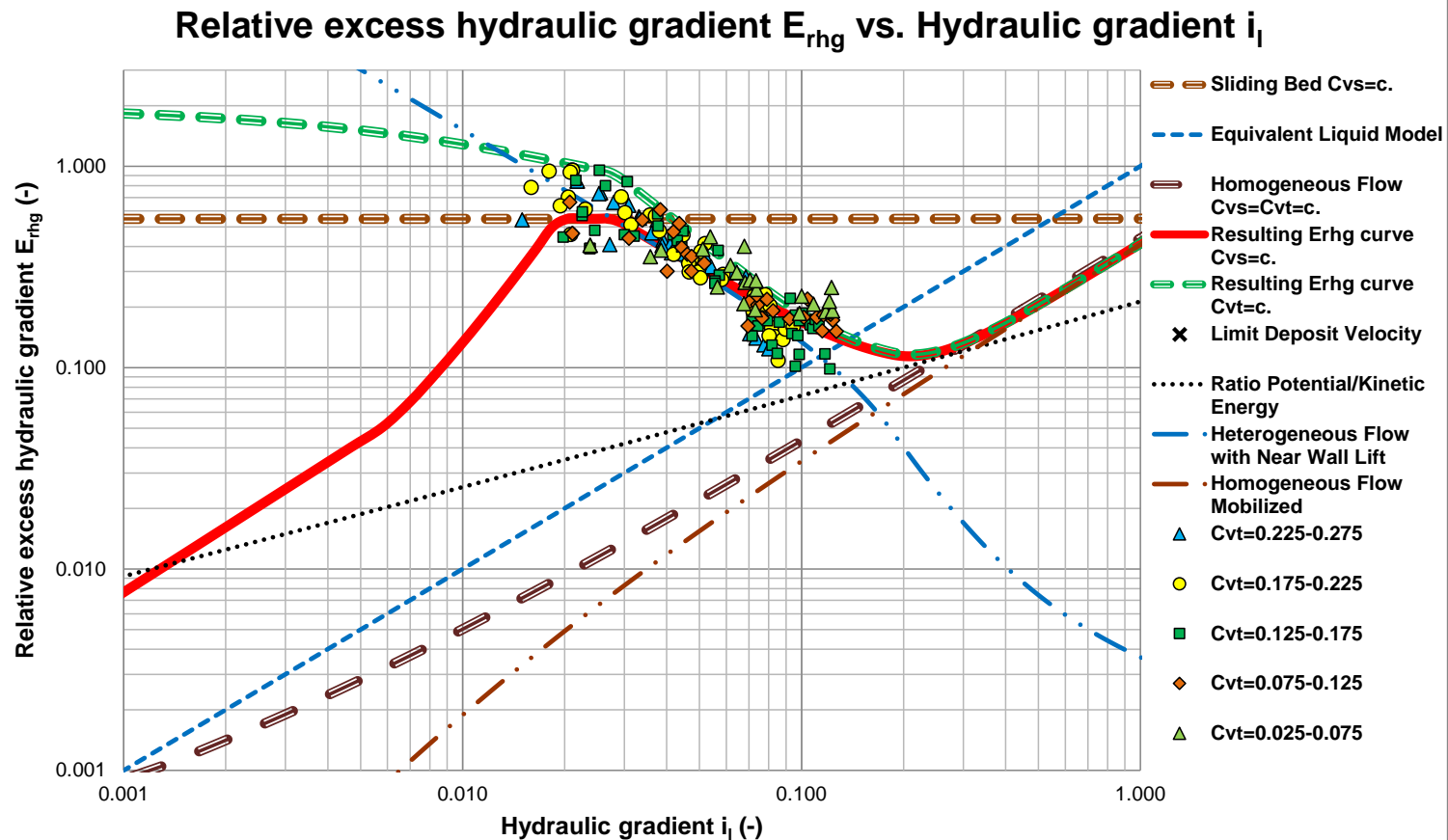


$D_p=0.1524 \text{ m}, d=1.500 \text{ mm}, R_{sd}=1.585, C_v=0.300, \mu=0.420$

$$i_l = \frac{\Delta p_l}{\rho_l \cdot g \cdot \Delta L} = \frac{\lambda_1 \cdot v_{ls}^2}{2 \cdot g \cdot D_p}$$

Hydraulic Gradient  
Relative Excess H.G.

$$E_{rhg} = \frac{i_m - i_l}{R_{sd} \cdot C_v}$$

Data from Yagi et al.,  $E_{rhg}-i_l$ 

$D_p=0.1552$  m,  $d=0.910$  mm,  $Rsd=1.585$ ,  $Cv=0.150$ ,  $\mu_{sf}=0.550$

Data looks more organized not depending on the volumetric concentration of the solids.

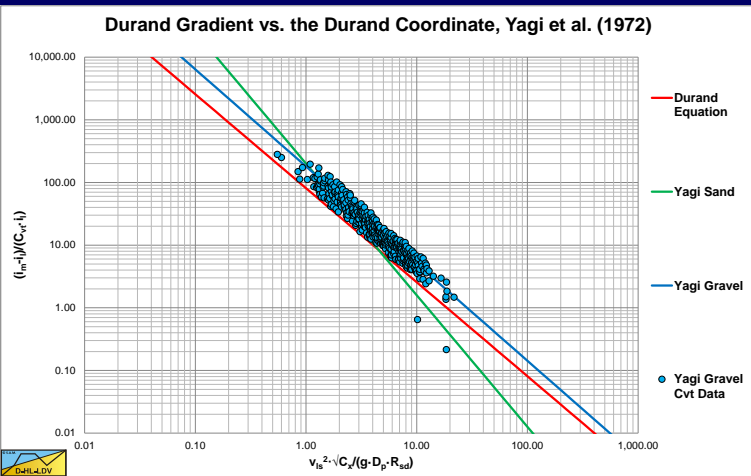
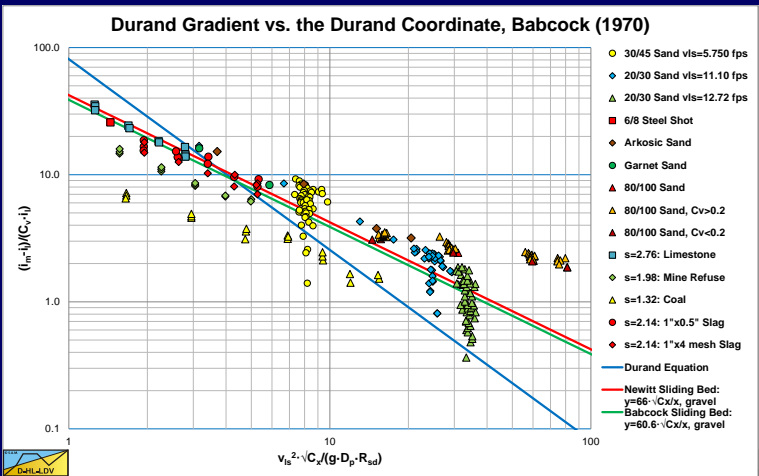
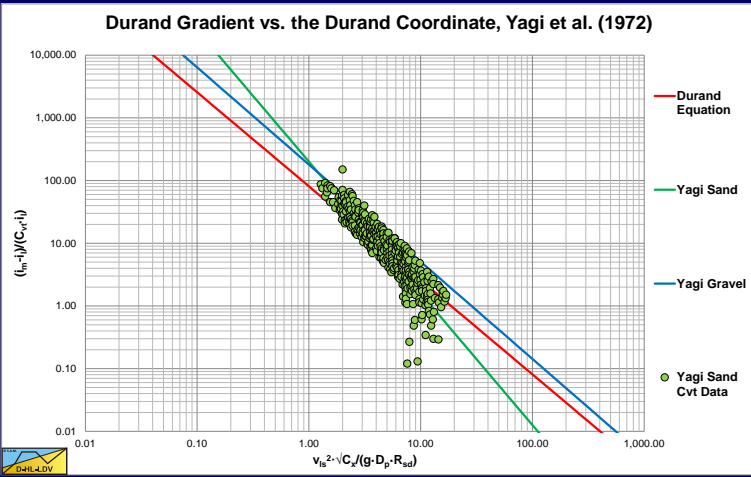
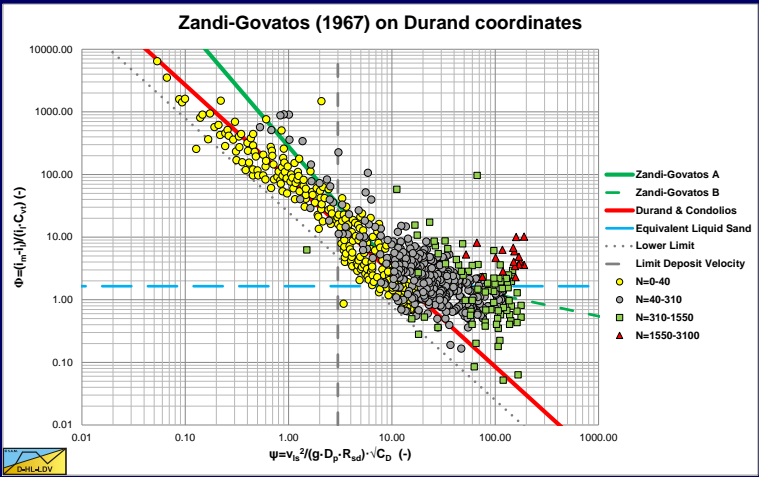
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## Existing Models

## Zandi & Govatos, Yagi et al. & Babcock





Existing Equations Depending on  $i_1$ 

$$\Delta p_m = \Delta p_1 \cdot (1 + \Phi \cdot C_{vt}) \quad \text{with:} \quad \Phi = \frac{i_m - i_1}{i_1 \cdot C_{vt}} = \frac{\Delta p_m - \Delta p_1}{\Delta p_1 \cdot C_{vt}}$$

Durand, Condolios & Gibert based on Froude numbers

$$\Phi = K \cdot \psi^{-3/2} = K \cdot \left( \frac{v_{ls}^2}{g \cdot D_p \cdot R_{sd}} \cdot \sqrt{C_x} \right)^{-3/2} \quad \text{with:} \quad K \approx 85$$

Newitt et al. based on potential energy losses

$$\Delta p_m = \Delta p_1 \cdot \left( 1 + K_1 \cdot (g \cdot D_p \cdot R_{sd}) \cdot v_t \cdot C_{vt} \cdot \left( \frac{1}{v_{ls}} \right)^3 \right) \quad K_1 = 1100$$

Jufin & Lopatin empirical large diameters

$$\Delta p_m = \Delta p_1 \cdot \left( 1 + 2 \cdot \left( \frac{v_{\min}}{v_{ls}} \right)^3 \right) \Rightarrow v_{\min} = 5.5 \cdot (C_{vt} \cdot \psi^* \cdot D_p)^{1/6}$$



Existing Equations Independent of  $i_1$ 

## Fuhrboter medium diameters

$$\Delta p_m = \Delta p_1 + \rho_l \cdot g \cdot \Delta L \cdot \frac{S_k}{v_{ls}} \cdot C_{vs}$$

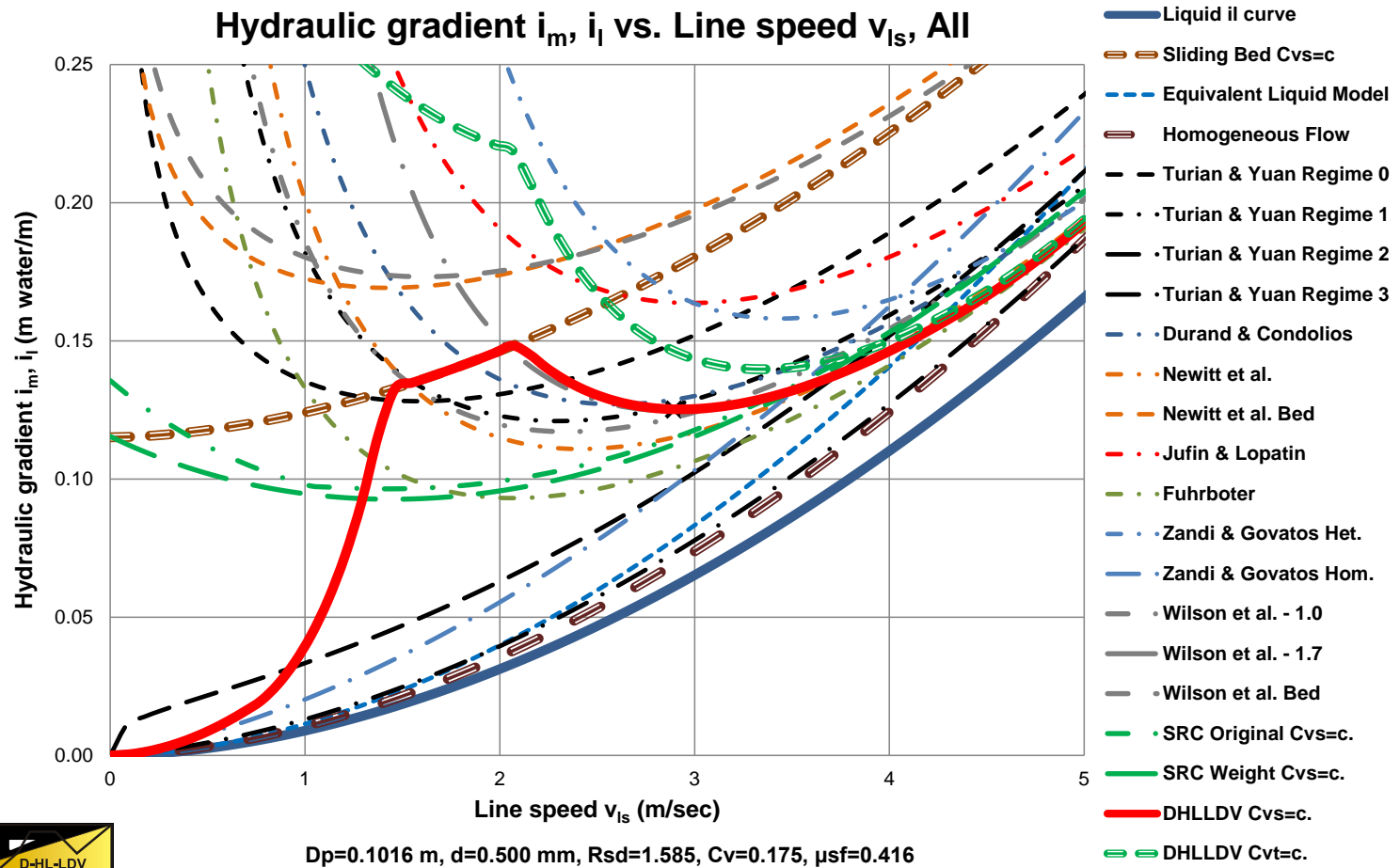
$$i_m - i_1 = \frac{S_k}{v_{ls}} \cdot C_{vs} \quad \Rightarrow \quad E_{rhg} = \frac{i_m - i_1}{R_{sd} \cdot C_{vs}} = \frac{S_k}{R_{sd} \cdot v_{ls}}$$

## Wilson heterogeneous empirical (Stratification Ratio)

$$\Delta p_m = \Delta p_1 + \frac{\mu_{sf}}{2} \cdot \rho_l \cdot g \cdot R_{sd} \cdot \Delta L \cdot \left( \frac{v_{50}}{v_{ls}} \right)^M \cdot C_{vt}$$

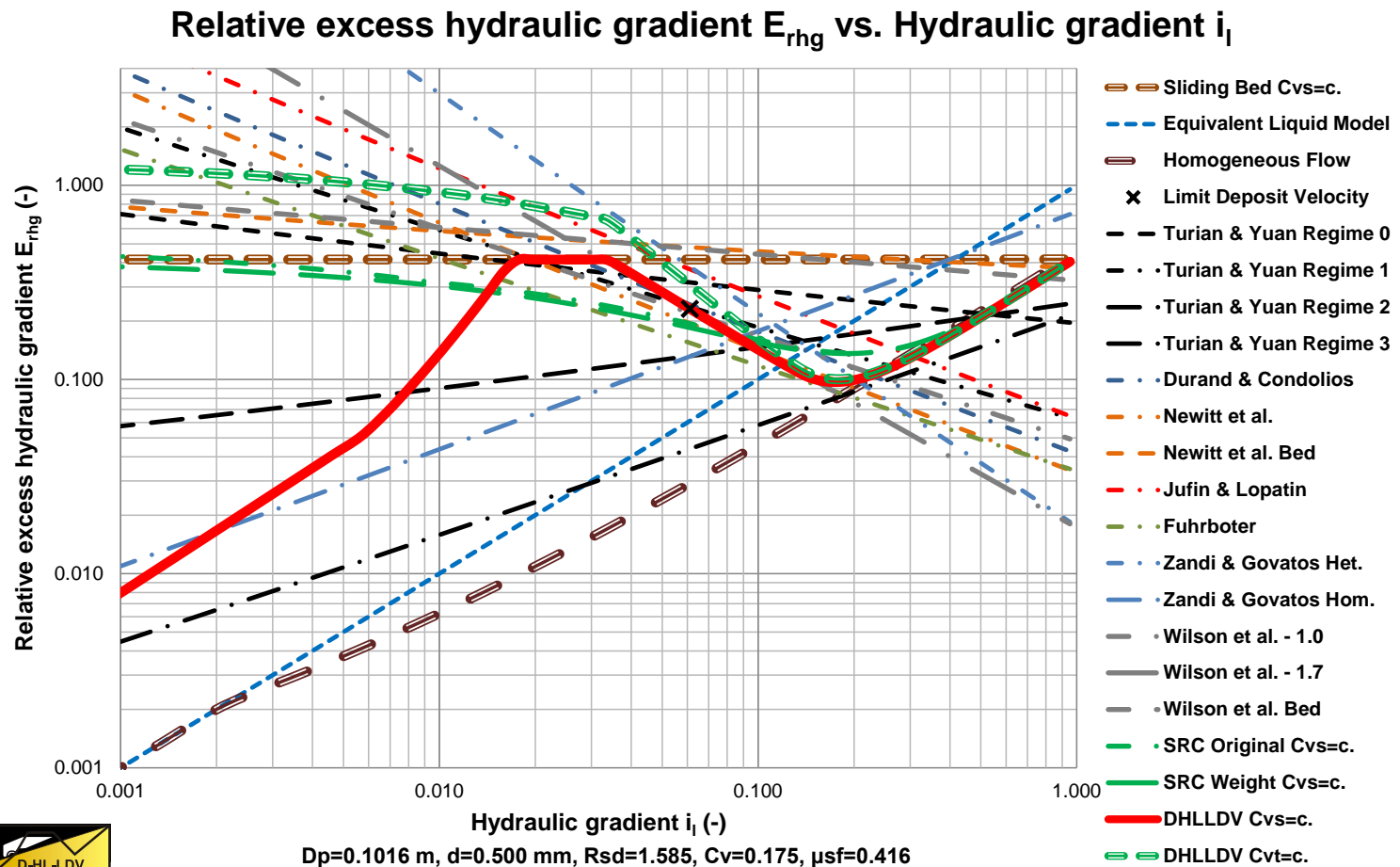
$$i_m - i_1 = \frac{\mu_{sf}}{2} \cdot R_{sd} \cdot \left( \frac{v_{50}}{v_{ls}} \right)^M \cdot C_{vt} \quad \Rightarrow \quad E_{rhg} = \frac{\mu_{sf}}{2} \cdot \left( \frac{v_{50}}{v_{ls}} \right)^M = R$$



22 Models  $i_m$ - $v_{ls}$  graph

For small pipe diameters the models are still “close”. For large diameter pipes the difference is much more.

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22 Models  $E_{rhg}$ - $i_1$  graph

This graph organizes the models better, but there is still a lot of difference between the models.

## Existing Equations Summary

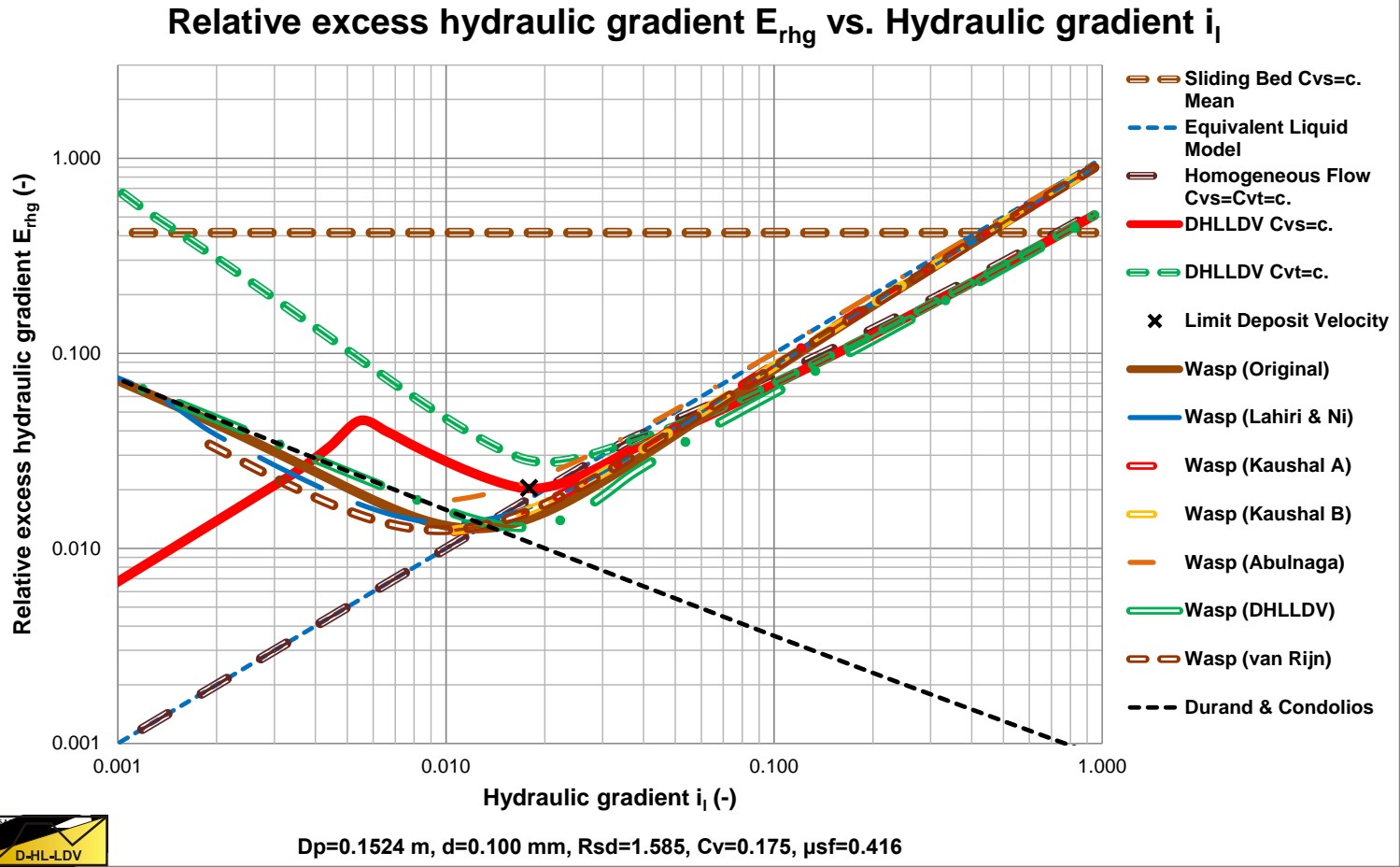
- All equations have the solids effect in just one term, limiting the possibilities to get a high correlation with experimental data.
- The first 3 equations multiply the solids effect with the Darcy Weisbach equation, making it dependent on the Darcy-Weisbach friction coefficient from the Moody diagram.
- The Wilson & Fuhrboter equations have an independent solids effect.
- All equations have a negligible solids effect at very high line speeds.
- The Wasp model has a solution for this by combining the Durand & Condolios model with the ELM.





## The Wasp Model

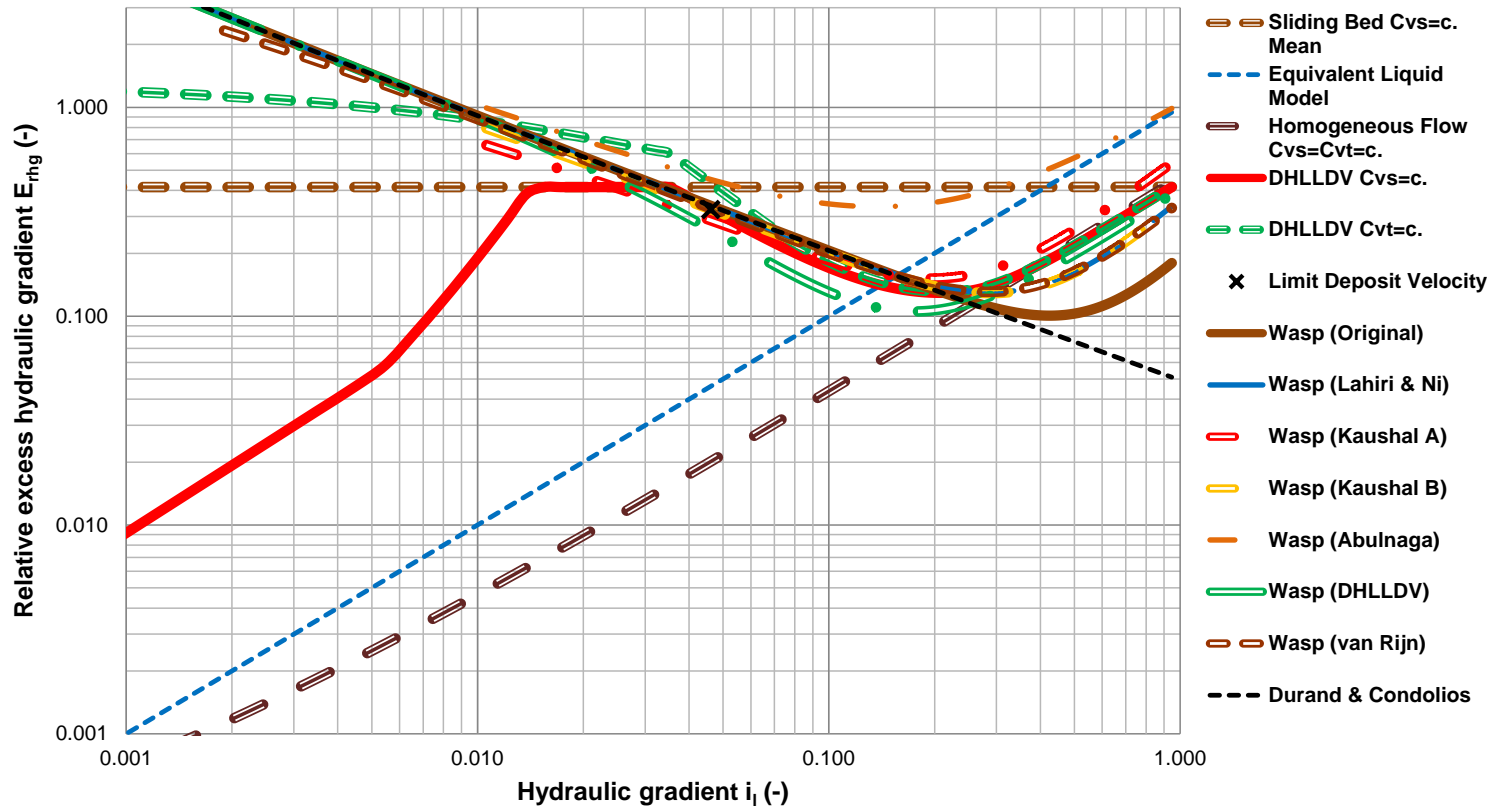
## The Wasp Model, Small Particle Diameter



# The Wasp Model, Large Particle Diameter



Relative excess hydraulic gradient  $E_{rhg}$  vs. Hydraulic gradient  $i_1$



$D_p=0.1524$  m,  $d=1.000$  mm,  $R_{sd}=1.585$ ,  $C_v=0.175$ ,  $\mu_{sf}=0.416$







## Transition Line Speed Heterogeneous - Homogeneous

# The Transition Line Speed Heterogeneous-Homogeneous

## Problem definition:

For slurry transport in general and specifically in dredging, there are many models. But how to decide which model to use in which situation, or, when are specific models valid especially in the heterogeneous flow regime.

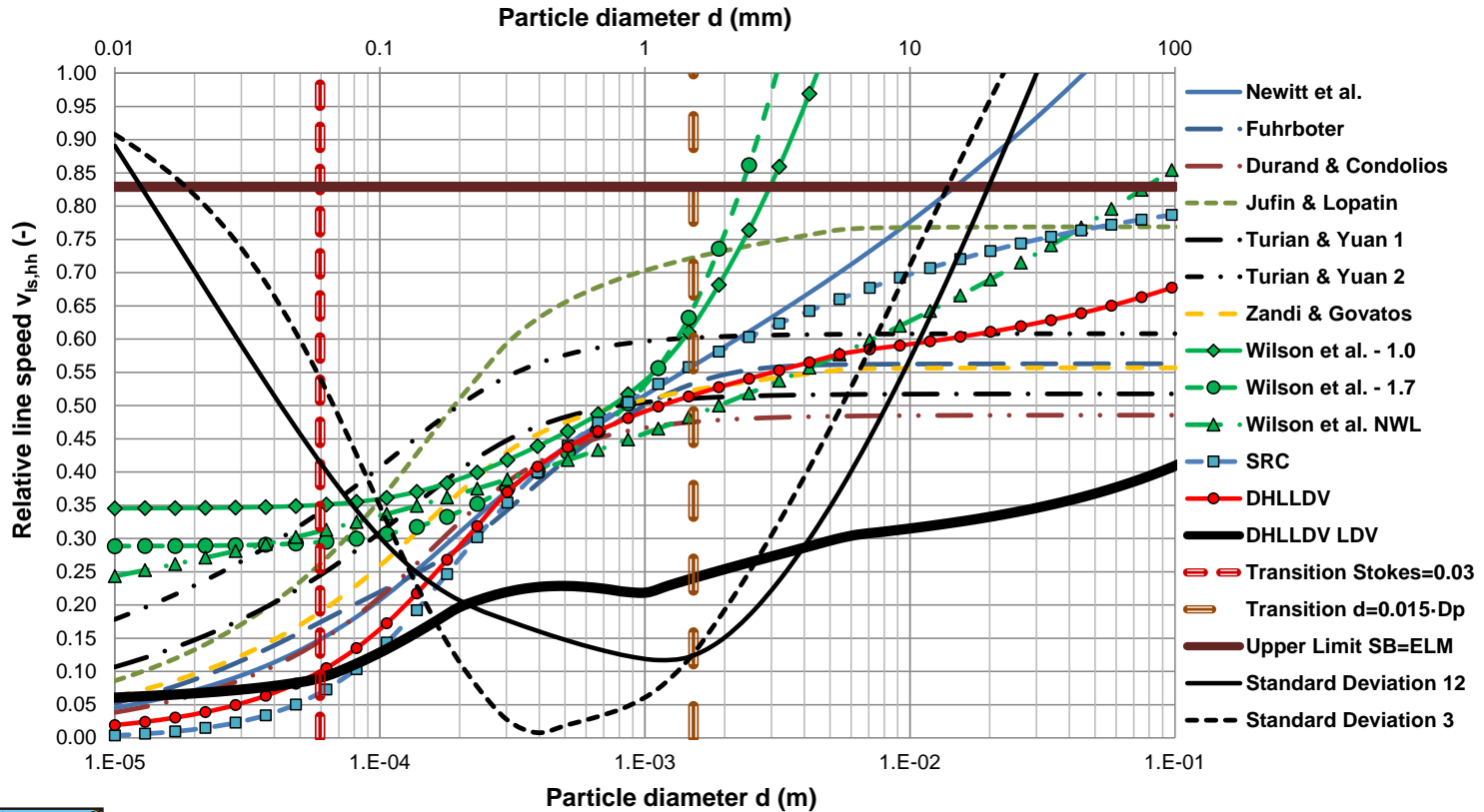
## Solution:

The transition line speed of the heterogeneous flow regime with the homogeneous flow regime is a good indicator and limits the number of graphs.



## Relative Transition Line Speed $D_p=0.1016 \text{ m}, C_{vs}=0.05$

### Transition Heterogeneous - Homogeneous



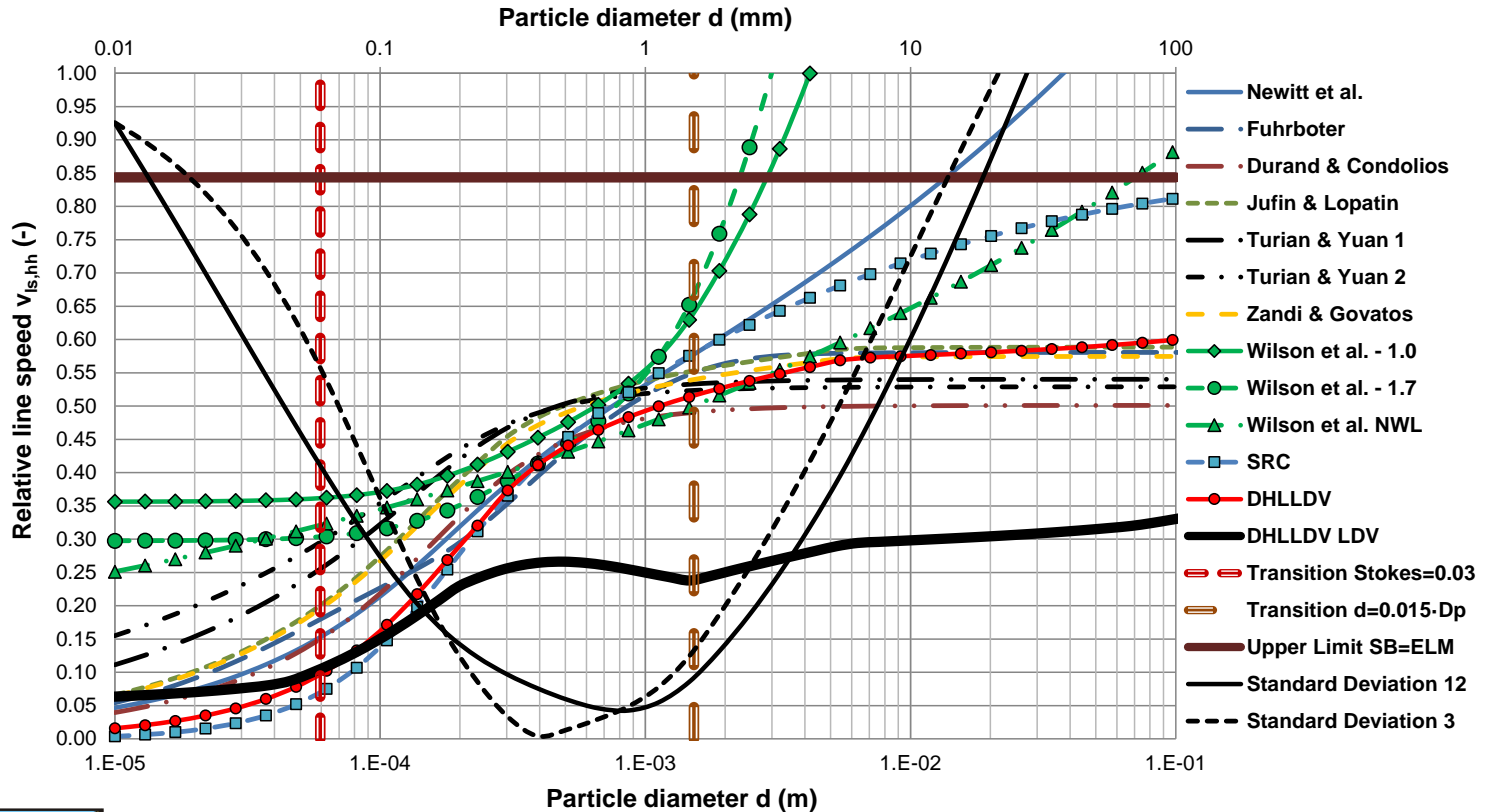
$D_p=0.1016 \text{ m}, R_{sd}=1.585, C_{vs}=0.050, \mu_{sf}=0.416$

$v_{ls, hh, max}=9.8 \text{ m/sec}$



## Relative Transition Line Speed $D_p=0.1016 \text{ m}, C_{vs}=0.30$

### Transition Heterogeneous - Homogeneous



$D_p=0.1016 \text{ m}, R_{sd}=1.585, C_{vs}=0.300, \mu_{sf}=0.416$

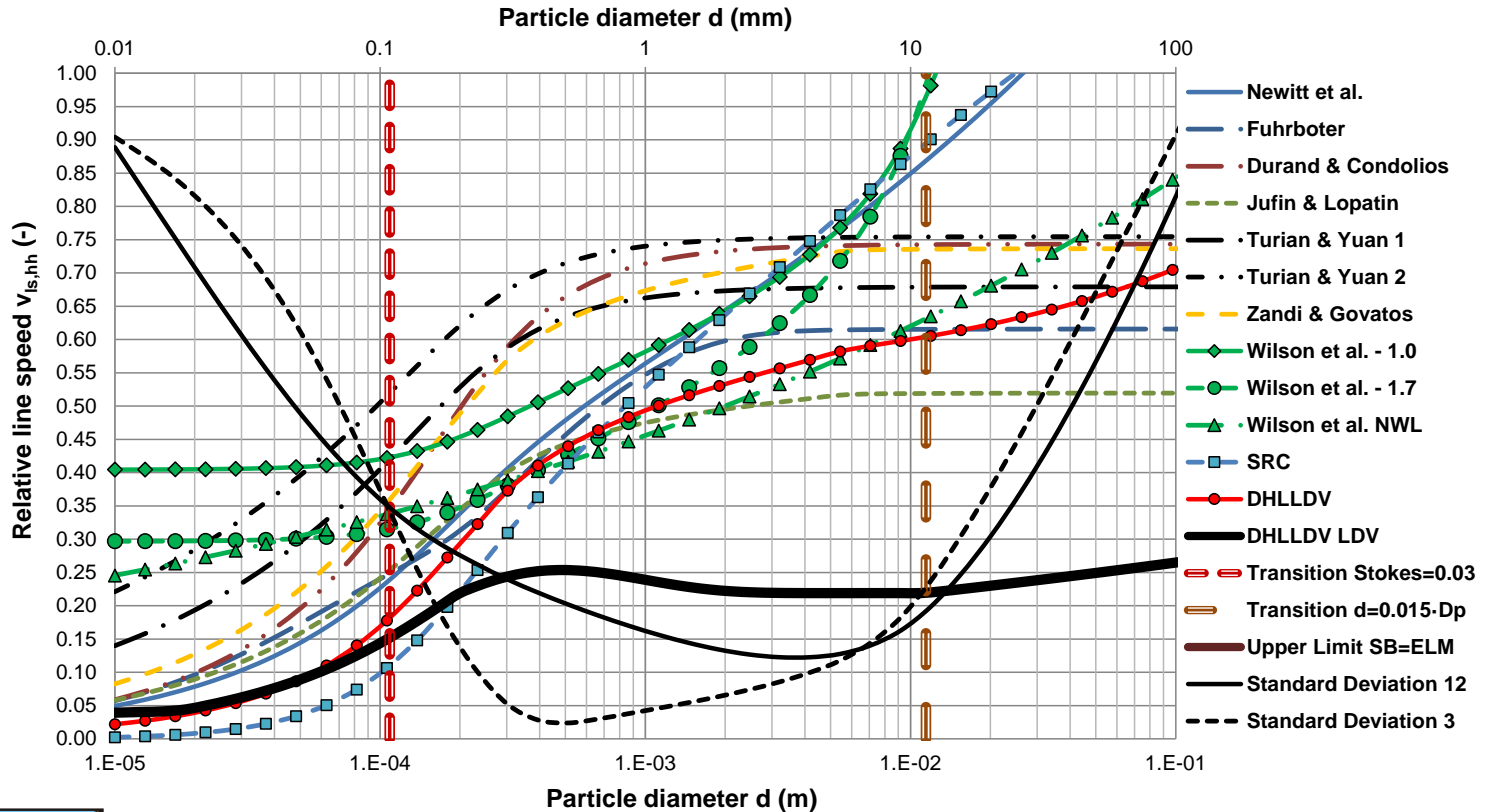
$v_{ls,hh,max}=9.5 \text{ m/sec}$



## Relative Transition Line Speed

$D_p = 0.7620 \text{ m}, C_{vs} = 0.05$

### Transition Heterogeneous - Homogeneous



$D_p = 0.7620 \text{ m}, R_{sd} = 1.585, C_{vs} = 0.050, \mu_{sf} = 0.416$

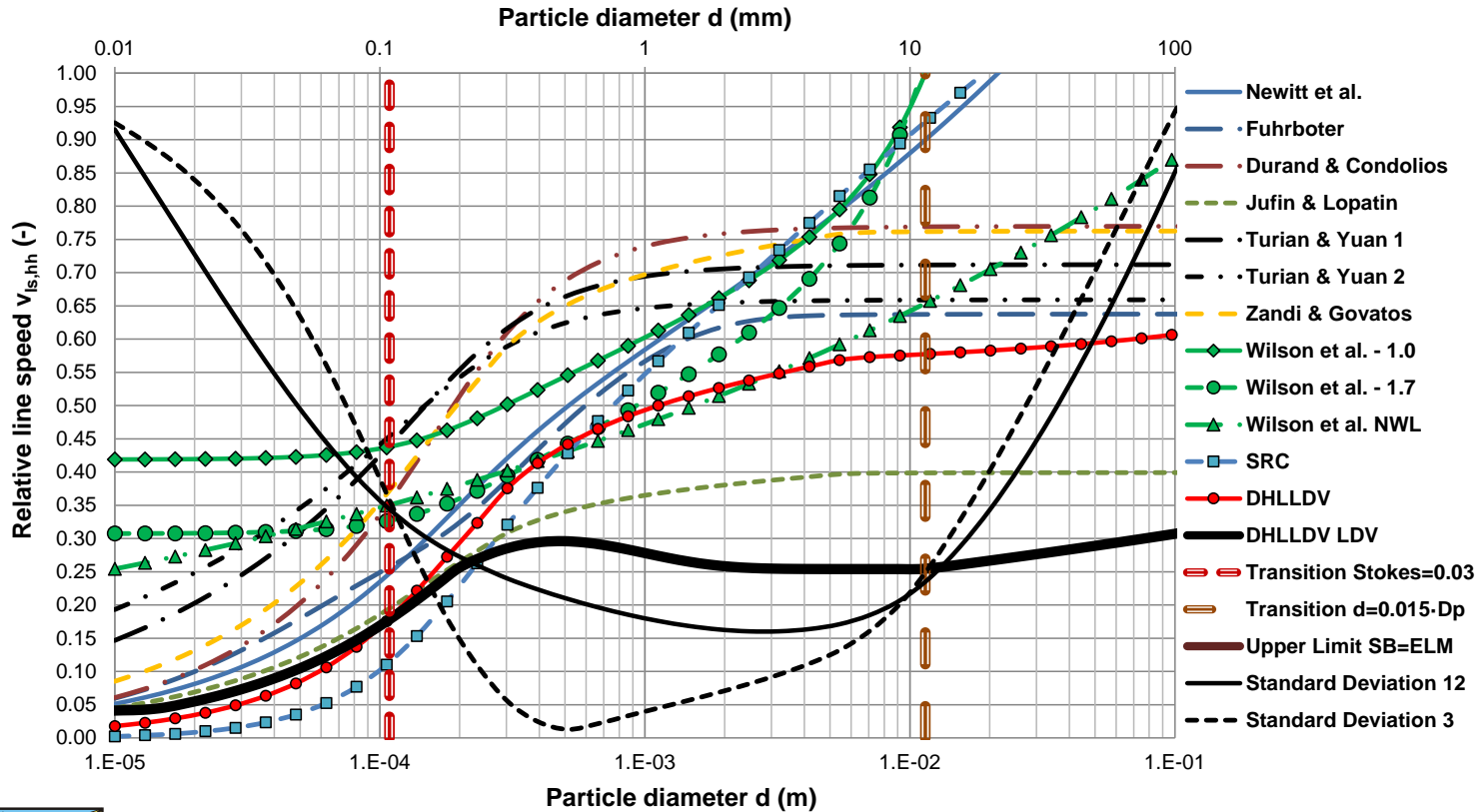
$v_{ls,hh,max} = 20.3 \text{ m/sec}$



## Relative Transition Line Speed

$D_p = 0.7620 \text{ m}, C_{vs} = 0.30$

### Transition Heterogeneous - Homogeneous



$D_p = 0.7620 \text{ m}, R_{sd} = 1.585, C_{vs} = 0.300, \mu_{sf} = 0.416$

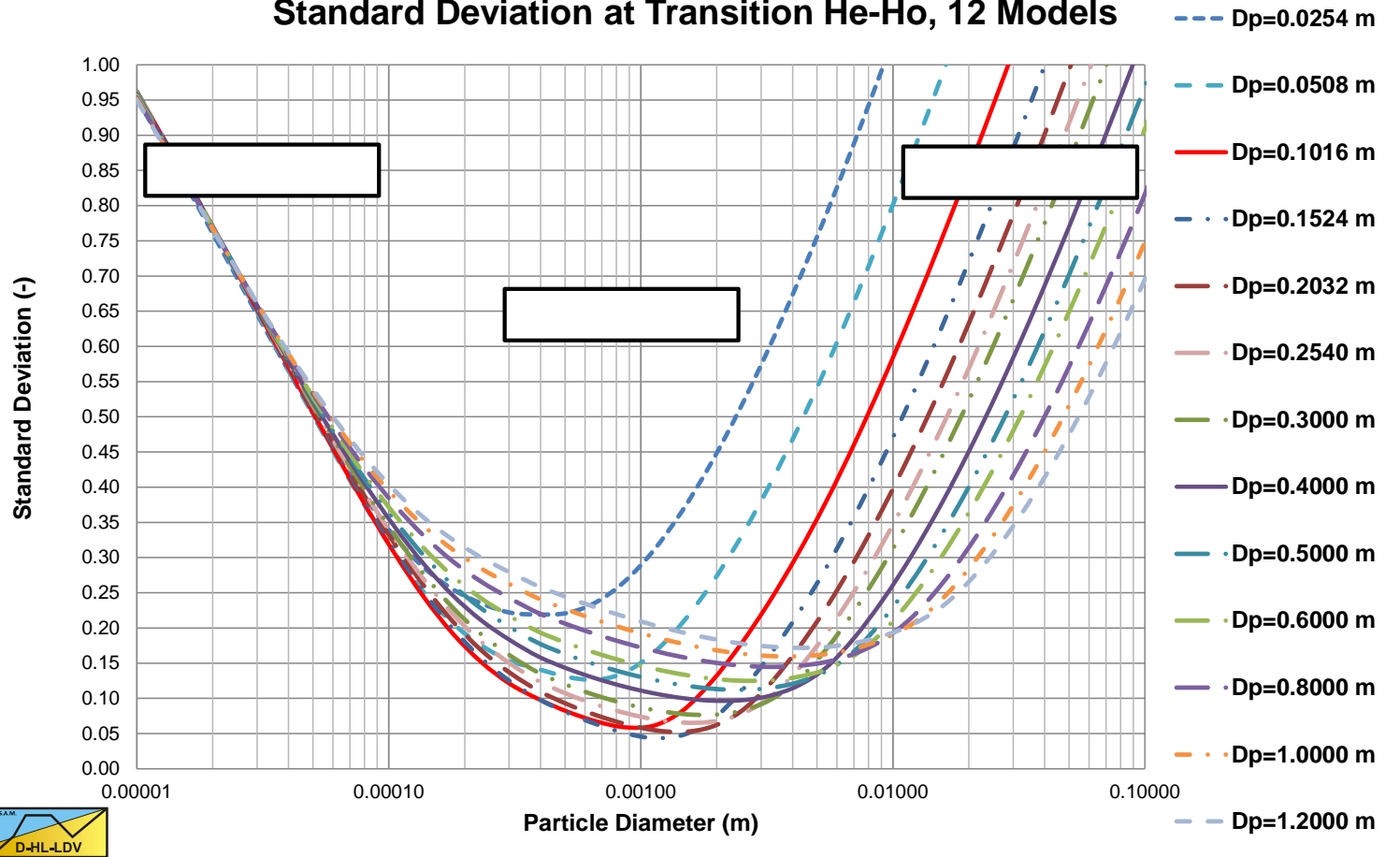
$v_{ls, hh, max} = 19.6 \text{ m/sec}$



## Standard Deviation 12 Models



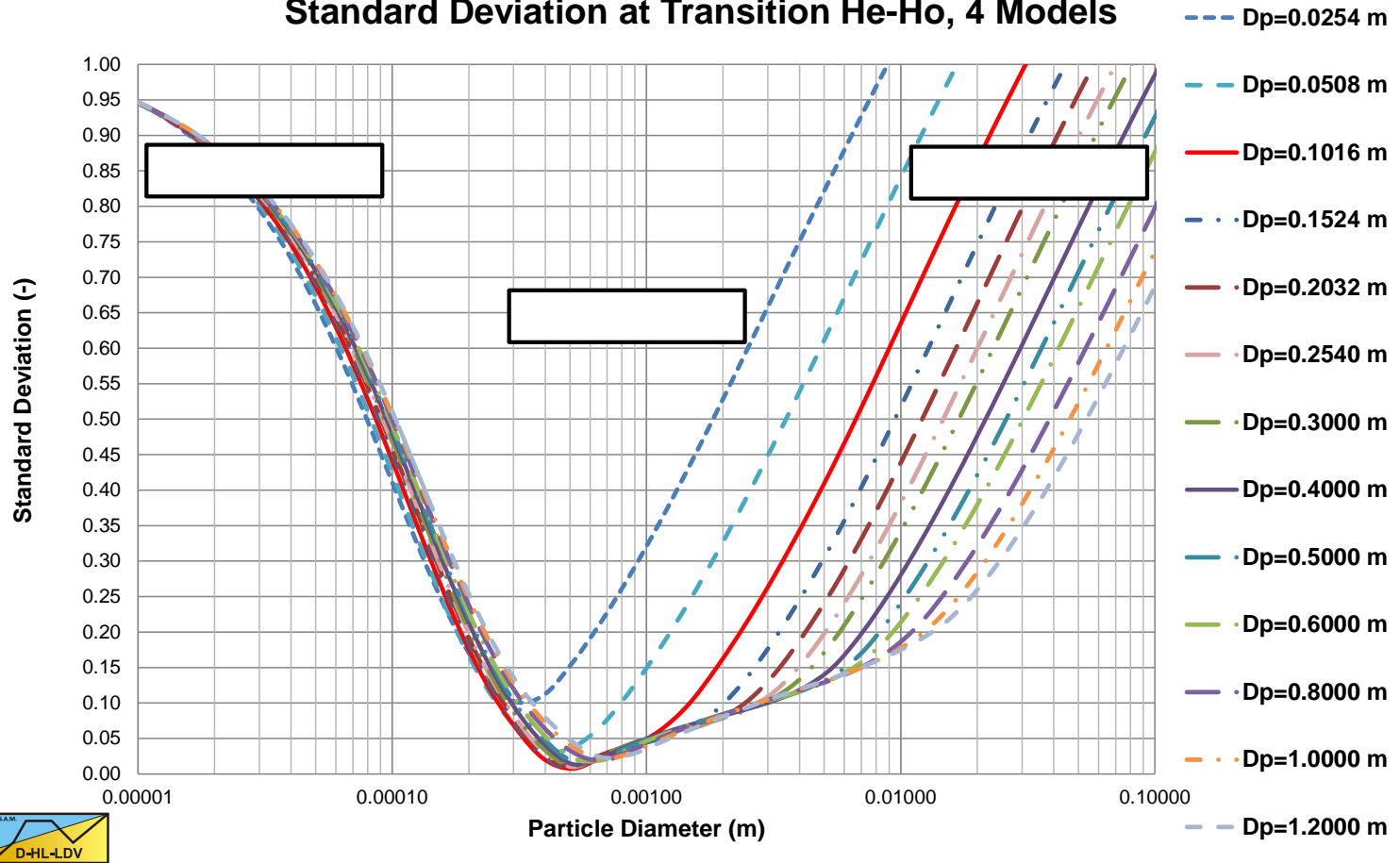
### Standard Deviation at Transition He-Ho, 12 Models



## Standard Deviation DHLLDV-Wilson/SRC



Standard Deviation at Transition He-Ho, 4 Models







## The Limit Deposit Velocity

## The Limit Deposit Velocity

### Problem definition:

For slurry transport in general and specifically in dredging, there is a critical velocity, the LDV. Operations should be above the LDV to avoid plugging the line. Which model to use to determine the LDV.

### Solution:

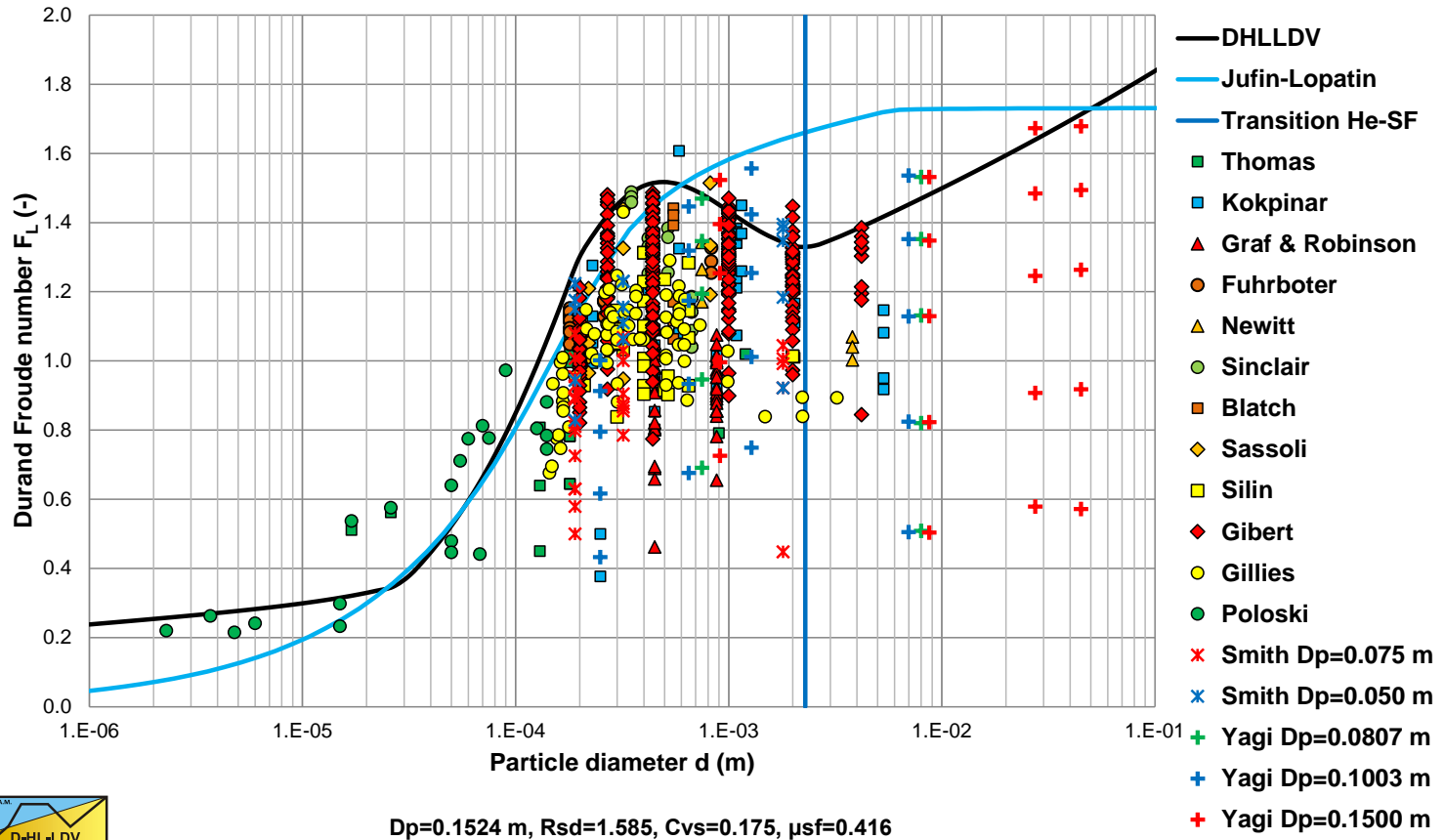
Using the Durand & Condolios Froude number  $F_L$  gives a good (dimensionless) indication of the LDV.





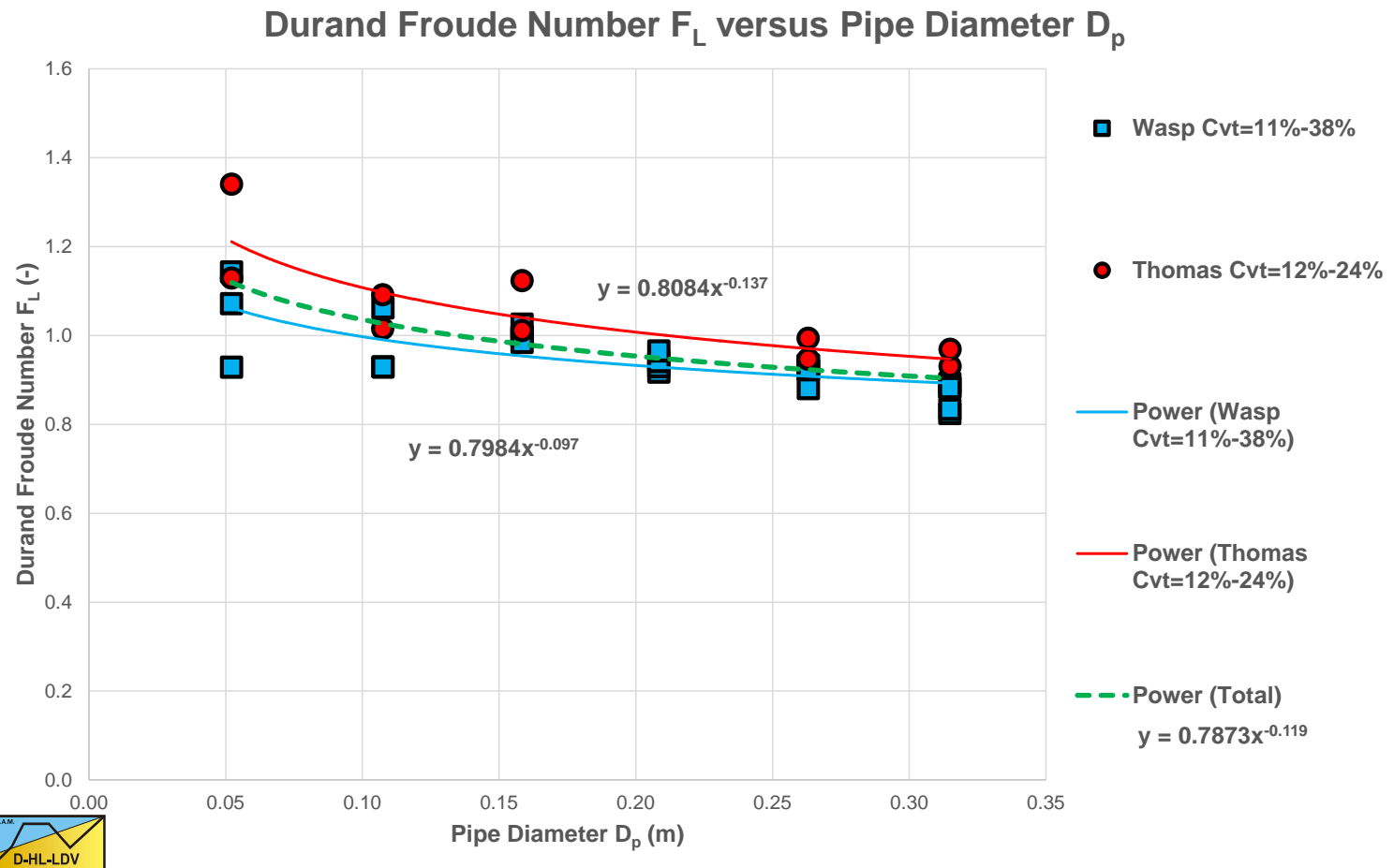
## Experiments

Durand Froude number  $F_L$  (-) vs. Particle diameter  $d$  (m)



$$F_L = \frac{v_{ls,ldv}}{\sqrt{2 \cdot g \cdot R_{sd} \cdot D_p}} = \frac{LDV}{\sqrt{2 \cdot g \cdot R_{sd} \cdot D_p}}$$

# Influence of Pipe Diameter

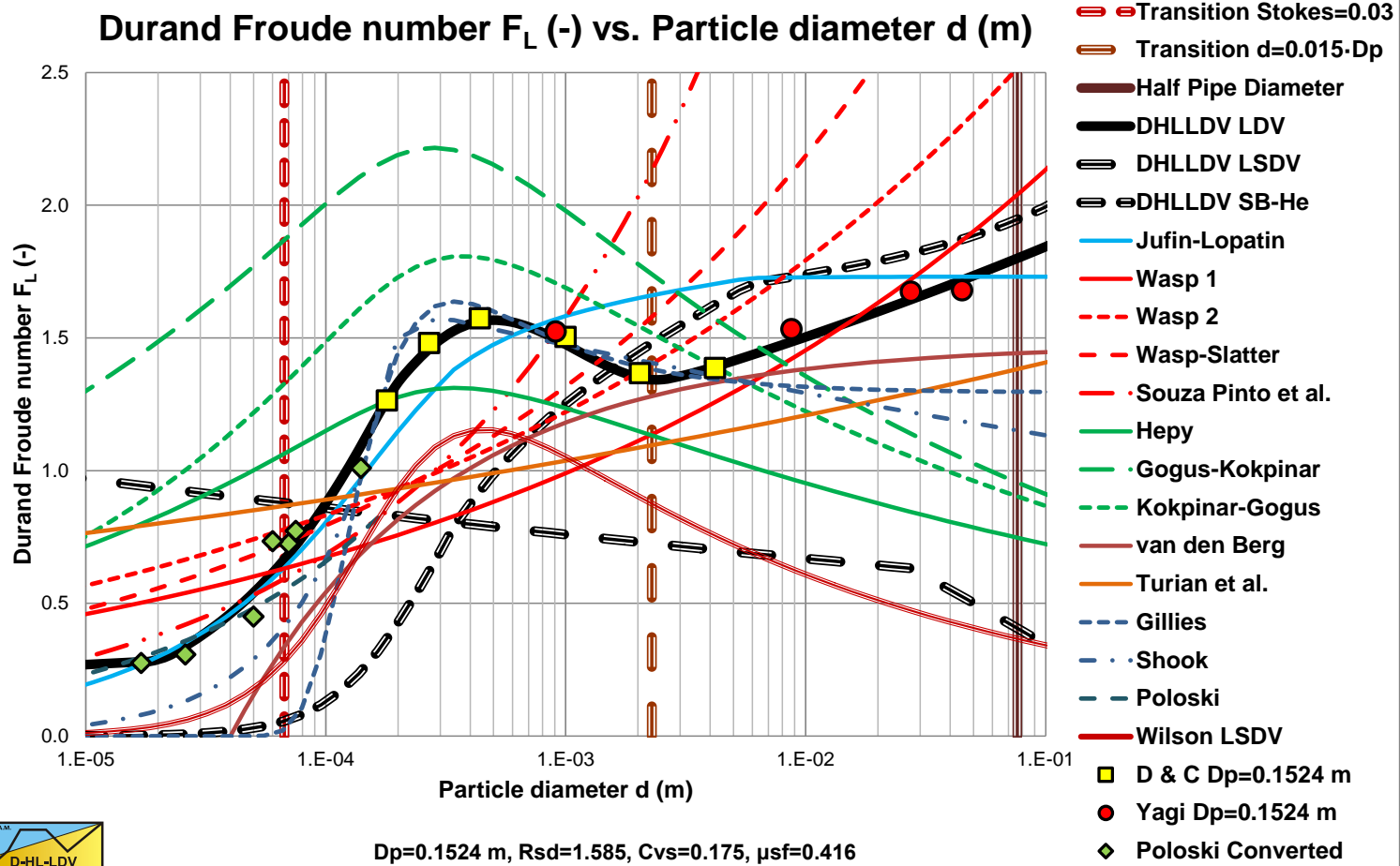


Thomas (1979) and Wasp et al. (1977)

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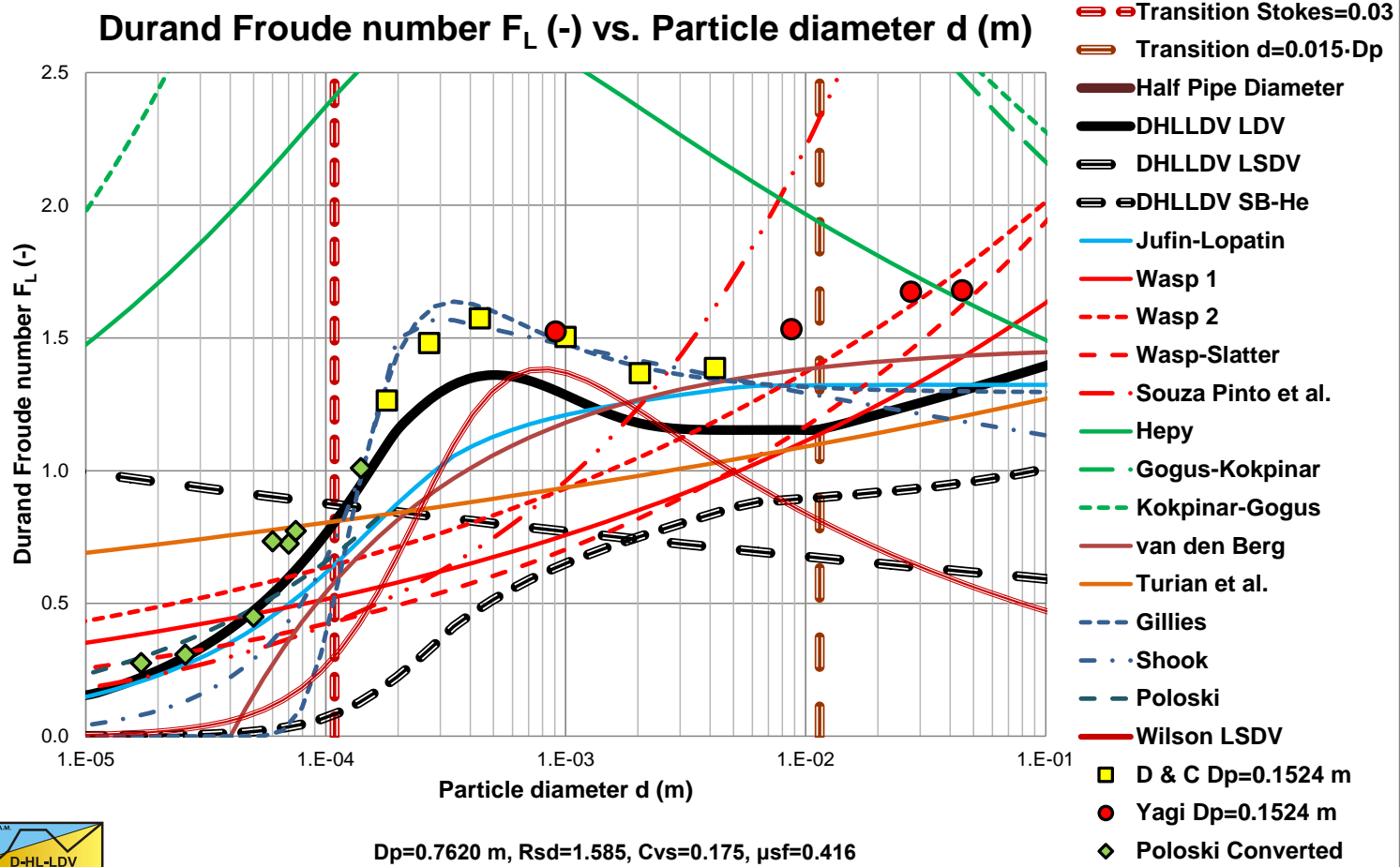


## 15 Models, $D_p=0.1524$ m





## 15 Models, $D_p=0.7620$ m



## Conclusions

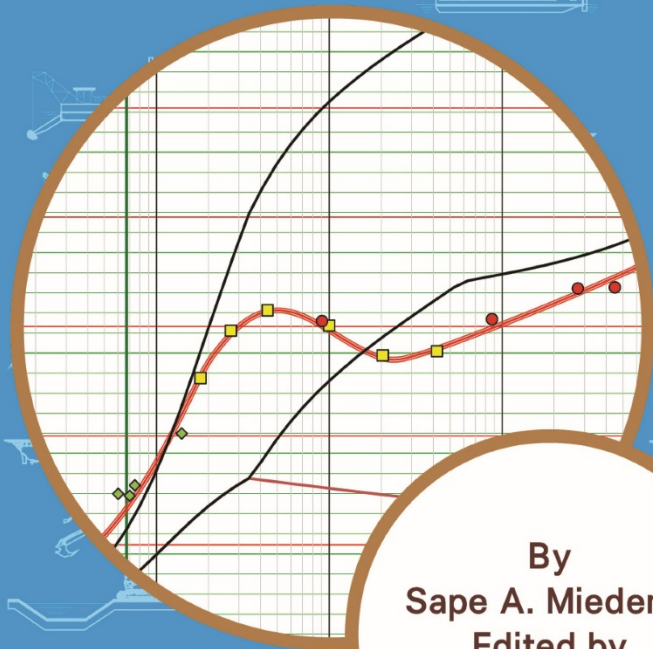
- The transition line speed of the heterogeneous flow regime with the homogeneous flow regime is a good indicator for comparing different head loss models.
- For pipe diameters near 4-6 inch most models perform the same. For smaller and larger pipe diameters the different models deviate.
- Based on numerous experimental data, the Wilson et al., the SRC and the DHLLDV models are the most reliable over a wide range of pipe and particle diameters.
- The LSDV and the LDV describe different physics and cannot be compared.
- The Durand & Condolios Froude number gives a good indication of the LDV with  $LDV=c \cdot D_p^{0.4}$ .





# SLURRY TRANSPORT

Fundamentals, A Historical Overview  
& The Delft Head Loss & Limit  
Deposit Velocity Framework



By  
Sape A. Miedema  
Edited by  
Robert C. Ramsdell





# Questions?