THE CUTTING OF WATER SATURATED SAND, THE FINAL SOLUTION

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ABSTRACT

When cutting water saturated sand, as is done in dredging, agriculture and soil movement in general, the process is dominated by the phenomenon of dilatancy. Based on pore pressure calculations and the equilibrium of horizontal and vertical forces, equations can be derived to predict the cutting forces. The derivation of this model has been described extensively in previous papers by Miedema et al. (1983-2005). In the equations derived, the denominator contains the sine of the sum of the 4 angles involved, the cutting angle α, the shear angle β, the angle of internal friction φ and the soil interface friction angle δ. So when the sum of these 4 angles approaches 180° the sine will become zero and the cutting forces become infinite. When the sum of these 4 angles is greater then 180° the sine becomes negative and so do the cutting forces. Since this does not occur in reality, nature must have chosen a different mechanism for the case where the sum of these 4 angles approaches 180°.

Hettiaratchi and Reece, (1975) found a mechanism which they called boundary wedges for dry soil. At large cutting angles a triangular wedge will exist in front of the blade, not moving relative to the blade. This wedge acts as a blade with a smaller blade angle. In fact, this reduces the sum of the 4 angles involved to a value much smaller than 180°. The existence of a dead zone (wedge) in front of the blade when cutting at large cutting angles will affect the value and distribution of vacuum water pressure on the interface. He, (1998), proved experimentally that also in water saturated sand at large cutting angles a wedge will occur.
The main questions however are; at which blade angle does a wedge start to occur, how does this depend on the soil mechanical, geometrical and operational parameters and what will be the geometry of the wedge.
Based on the equilibrium of moments a solution is found to answer these questions.

INTRODUCTION

In the last decennia extensive research has been carried out into the cutting of water saturated sand. In the cutting of water-saturated sand, the phenomenon of dilatation plays an important role. In fact the effects of gravity, inertia, cohesion and adhesion can be neglected at cutting speeds in the range of 0.5 – 10 m/s. In the cutting equations, as published by Miedema, there is a division by the sine of the sum of the blade angle, the shear angle, the angle of internal friction and the soil/interface friction angle. When the sum of these angle approaches 180°, a division by zero is the result, resulting in infinite cutting forces. This may occur for example for α=80°, β=30°, ϕ=40° and δ=30°. When this sum is greater then 180 degrees, the cutting forces become negative. It is obvious that this cannot be the case in reality and that nature will look for another cutting mechanism. Hettiaratchi and Reece, (1975) found a mechanism which they called boundary wedges for dry soil. At large cutting angles a triangular wedge will exist in front of the blade, not moving relative to the blade. This wedge acts as a blade with a smaller blade angle. In fact, this reduces the sum of the 4 angles mentioned before to a value much smaller than 180°. The existence of a dead zone (wedge) in front of the blade when cutting at large cutting angles will affect the value and distribution of vacuum water pressure on the interface. He, (1998), proved experimentally that also in water saturated sand at large cutting angles a wedge will occur. A series of tests with rake angles 90, 105 and 120 degrees under fully saturated and densely compacted sand condition was performed by Jisong He at the Dredging Technology section of Delft University of Technology. The experimental results showed that the failure pattern with large rake angles is quite different from that with small rake angles. For large rake angles a dead zone is formed in front of the blade but not for small rake angles. In the tests he carried out, both a video camera and film camera were used to capture the failure pattern. The video camera was fixed on the frame which is mounted on the main carriage, translates with the same velocity as the testing cutting blade. Shown in the static slide of the video record, as in Figure 1, the boundary wedges exist during the cutting test.

Although the number of experiments published is limited, his research is valuable as a starting point to predict the shape of the wedge. At small cutting angles the cutting forces are determined by the horizontal and vertical force

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equilibrium equations of the sand cut in front of the blade. These equations contain 3 unknowns, so a third equation/condition had to be found. The principle of minimum energy is used as a third condition to solve the 3 unknowns. This has proved to give very satisfactory results finding the shear angle and the horizontal and vertical cutting forces at small cutting angles. At large cutting angles, a 4th unknown exists, the wedge angle or virtual blade angle. This means that a 4th equation/condition must be found in order to determine the wedge angle. There are 3 possible conditions that can be used: The principle of minimum energy, The circle of Mohr, The equilibrium of moments of the wedge. In fact, there is also a 5th unknown, the mobilized friction on the blade.

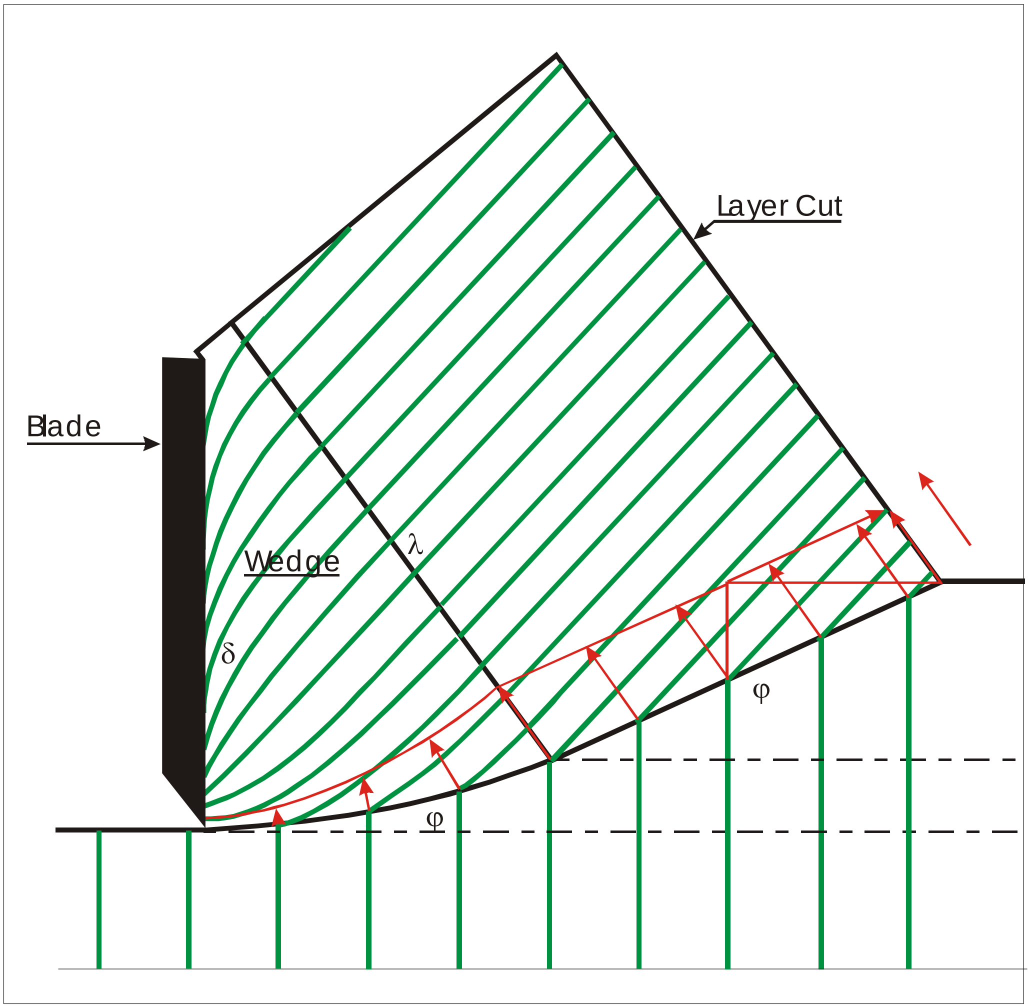

Figure 1: The static/dynamic wedge.

CALCULATION OF THE CUTTING FORCES

Because of the velocity distribution in the wedge, the internal friction angle between the wedge and the layer cut λ, does not have to be fully mobilized. This results in a set of modified cutting equations.
The forces that act on the blade during the cutting of soil, are transmitted on the blade through grain stresses and water pressures from wedge and cut soil. The forces on the cut layer are shown in fig. 2-4. These forces are:

1. Normal stress force N_2 between the wedge and the layer cut.
2. Shear stress force S_2 as a result of the internal friction of the sand between the wedge and cut layer.
3. Water pressure difference force W_2 between the wedge and cut layer resulting from p_2.

4. Normal stress force N_1 between the cut layer and ground floor.
5. Shear stress force S_1 as a result of the internal friction of the sand between the cut layer and ground floor.
6. Water pressure difference force W_1 between the cut layer and ground floor resulting from p_1.

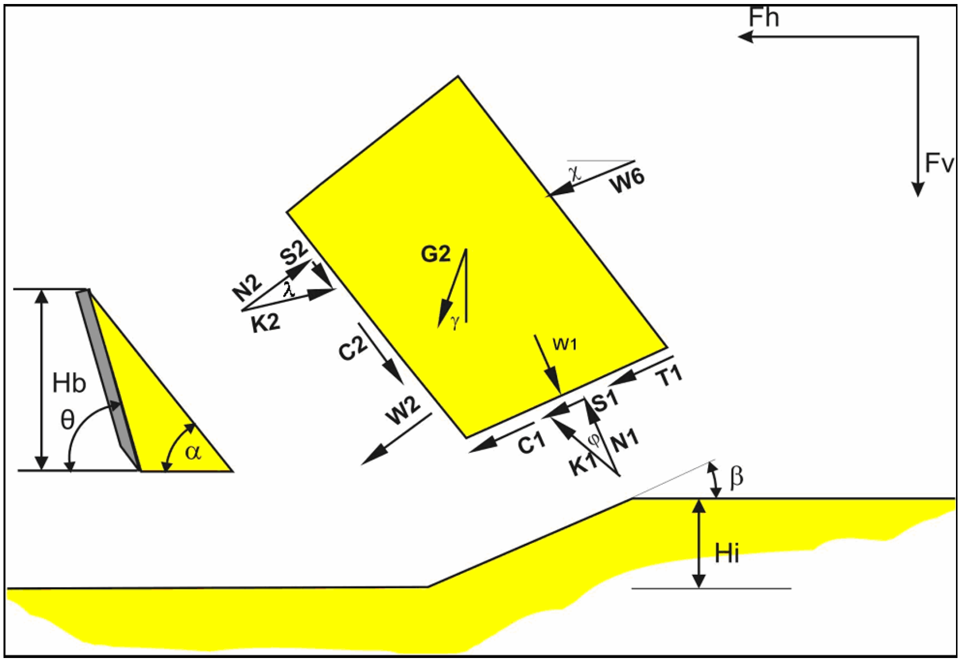

Figure 2: The forces on the layer cut.

If a wedge is considered like it is shown in Figure 2, the following equations can be derived for the forces acting on the wedge:

$$K_2 = \frac{1}{\sin(\alpha+\beta+\varphi+\lambda)}[W_2 \sin(\alpha+\beta+\varphi) + W_1 \sin\varphi] \tag{1}$$

The horizontal force F_h.

$$F_h = -W_2 \sin\alpha + K_2 \sin(\alpha+\lambda) \tag{2}$$

The vertical force F_v.

$$F_v = -W_2 \cos\alpha + K_2 \cos(\alpha+\lambda) \tag{3}$$

If there is no wedge, or the wedge angle α is equal to the blade angle θ, the following equations can be derived acting on the blade. These equations are almost identical to the equations 1, 2 and 3, with the provision that the wedge angle α is replaced by the blade angle θ and the internal friction angle λ acting on the interface between the soil cut and the wedge is replaced by the soil interface friction angle δ.

$$K_2 = \frac{1}{\sin(\theta+\beta+\varphi+\delta)}[W_2 \sin(\theta+\beta+\varphi) + W_1 \sin\varphi] \tag{4}$$

The horizontal force F_h.

$$F_h = -W_2 \sin\theta + K_2 \sin(\theta + \delta) \tag{5}$$

The vertical force F_v.

$$F_v = -W_2 \cos\theta + K_2 \cos(\theta + \delta) \tag{6}$$

The forces on the wedge layer are shown in Figure 3.

These forces are:

1. The earlier mentioned forces N_2, S_2 and W_2.
2. Water under-pressures on the blade p_4 , resulting in the force W_4.
3. Normal stress, resulting in the force N_4.
4. Shear stress as a result of the soil/steel friction, S_4.
5. Normal stress force N_3 between the wedge and ground floor.
6. Shear stress force S_3 as a result of the internal friction of the sand between the wedge and ground floor.
7. Water pressure difference force W_3 between the wedge and ground floor resulting from p_3.

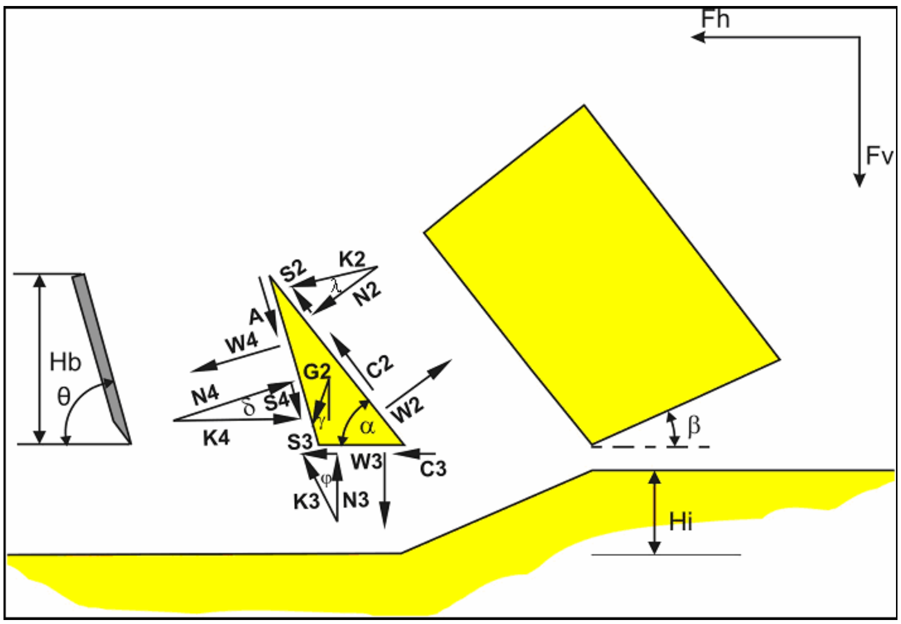

Figure 3: The forces on the wedge.

$$K_4 = \frac{1}{\sin(\theta + \delta + \varphi)}[W_3 \sin\varphi + W_4 \sin(\theta + \varphi) - W_2 \sin(\alpha + \varphi) + K_2 \sin(\alpha + \lambda + \varphi)] \tag{7}$$

The forces that act on the blade during the cutting of soil, are transmitted on the blade through grain stresses and water pressures. These forces are indicated in Figure 4.

The resulting water force on the blade W_4 can be determined theoretically. Since the grain force K_4 is known, forces on the blade can be calculated now. The following forces acting on the blade per unit width can be calculated:

The horizontal force F_h.

$$F_h = -W_4 \sin\theta + K_4 \sin(\theta+\delta) \quad (8)$$

The vertical force F_v.

$$F_v = -W_4 \cos\theta + K_4 \cos(\theta+\delta) \quad (9)$$

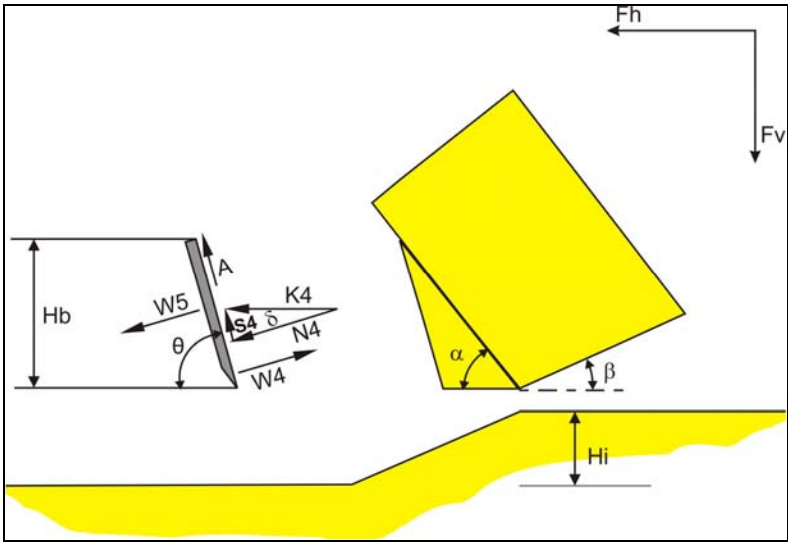

Figure 4: The forces on the blade.

If there is no cavitation the water pressures forces W_1, W_2, W_3 and W_4 can be written as:

$$W_1 = \frac{p_1 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i^2 \cdot b}{(a_1 \cdot k_1 + a_2 \cdot k_{max})} \quad (10)$$

$$W_2 = \frac{p_2 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i^2 \cdot b}{(a_1 \cdot k_1 + a_2 \cdot k_{max})} \quad (11)$$

$$W_3 = \frac{p_3 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i^2 \cdot b}{(a_1 \cdot k_1 + a_2 \cdot k_{max})} \quad (12)$$

$$W_4 = \frac{p_4 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i^2 \cdot b}{(a_1 \cdot k_1 + a_2 \cdot k_{max})} \quad (13)$$

Under a full cavitation situation the pore pressure reaches water vapor pressure $p = (z+10) \cdot \rho_w \cdot g$ with z as water depth. So the pore pressure forces become:

$$W_1 = (z+10) \cdot \rho_w \cdot g \cdot h_i / \sin\beta \quad (14)$$

$$W_2 = (z+10) \cdot \rho_w \cdot g \cdot h_b / \sin\alpha \quad (15)$$

$$W_3 = (z+10) \cdot \rho_w \cdot g \cdot h_b \cdot (1/\tan\alpha - 1/\tan\theta) \quad (16)$$

$$W_4 = (z+10) \cdot \rho_w \cdot g \cdot h_b / \sin\theta \quad (17)$$

On the wedge there is not only an equilibrium of horizontal and vertical forces, but there also has to be an equilibrium of moments. This equilibrium of course should exist around each point of the wedge, but for simplicity reasons the equilibrium equition has been derived around the edge of the blade, resulting in the following equation.

$$\begin{aligned} M_o &= e_4(N_4 - W_4)h_b \, {}^1\!/_{\sin\theta} + e_3(W_3 - N_3)h_b({}^1\!/_{\tan\alpha} - {}^1\!/_{\tan\theta}) \\ &- h_b({}^1\!/_{\tan\alpha} - {}^1\!/_{\tan\theta})S_2\sin\alpha + (h_b({}^1\!/_{\tan\alpha} - {}^1\!/_{\tan\theta})\cos\alpha - e_2 h_b \, {}^1\!/_{\sin\alpha}) * (N_2 - W_2) \end{aligned} \tag{20}$$

The resulting moment M_0 should be zero in the equilibrium situation. Equation 20 contains 3 new parameters e_2, e_3 and e_3 which correspond with the relative positions of the acting points of the forces on the 3 sides of the wedge. The parameter e_2 is the position of the acting point on the interface of the soil cut and the wedge, e_3 on the bottom of the wedge and e_4 on the blade. If an acting point is in the middle of a side the e value would be 0.5.

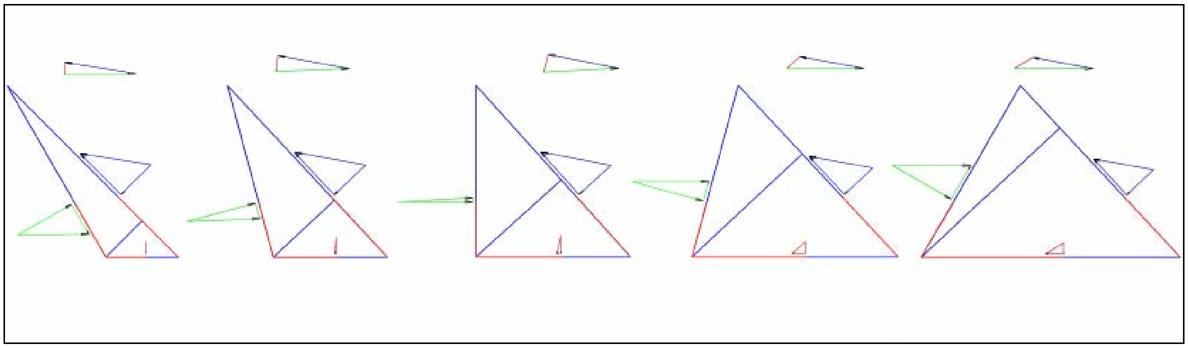

Figure 5: The forces on the wedges at 60°, 75°, 90°, 105° and 120° cutting angles.

Figure 5 shows the force triangles on the 3 sides of the wedges for cutting angles from 60 to 120 degrees. From the calculations it appeared that the pore pressures on interface between the soil cut and the wedge and in the shear plane do not change significantly when the blade angle changes. These pore pressures p_1 and p_2 resulting in W_1 and W_2 are determined by the shear angle β, the wedge angle α and other soil mechanical properties like the permeability.

The fact that the pore pressures do not change a lot also results in forces K_2, acting on the wedge that do not change a significantly, according to equations 1, 2 and 3. These forces are shown in Figure 5 on the right side of the wedges and the figure shows that these forces are almost equal for all blade angles. These forces are determined by the conventional theory as published by Miedema (1987). Figure 5 also shows that for the small blade angles the friction force on the wedge is directed downwards, while for the big blade angles this friction force is directed upwards.

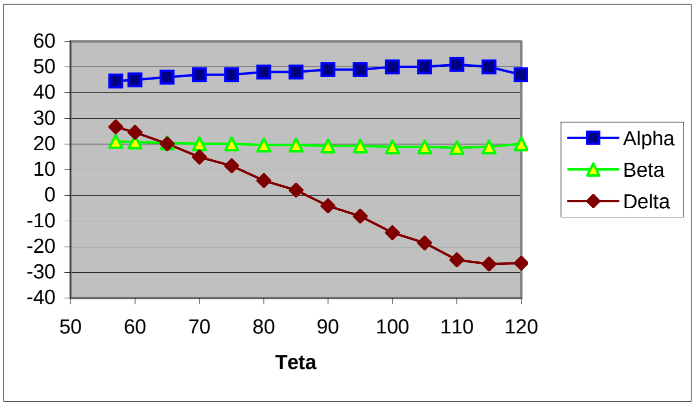

Figure 6: The wedge angle, shear angle and soil interface friction angle as a function of the blade angle.

Now the question is, what is the final solution for the cutting of water saturated sand at large cutting angles? From many calculations and an analysis of the laboratory research is described by He (1998), Miedema (2002) and Ma (2001), it appeared that the wedge can be considered a static wedge, although the sand inside the wedge still has velocity, the sand on the blade is not moving. The main problem in finding acceptable solutions was finding good values for the acting points on the 3 sides of the wedge, e_2, e_3 and e_4. If these values are choosen right, solutions exist based on the equilibrium of moments, but if they are chosen wrongly, no solution will be found. So the choice of these parameters is very critical. The statement that the sand on the blade is not moving is based on two things, first of all if the sand is moving with respect to the blade, the soil interface friction is fully mobilized and the bottom of the wedge requires to have a small angle with respect to the horizontal in order to make a flow of sand possible. This results in much bigger cutting forces, while often no solution can be found or unreasonable values for e_2, e_3 and e_4 have to be used to find a solution.

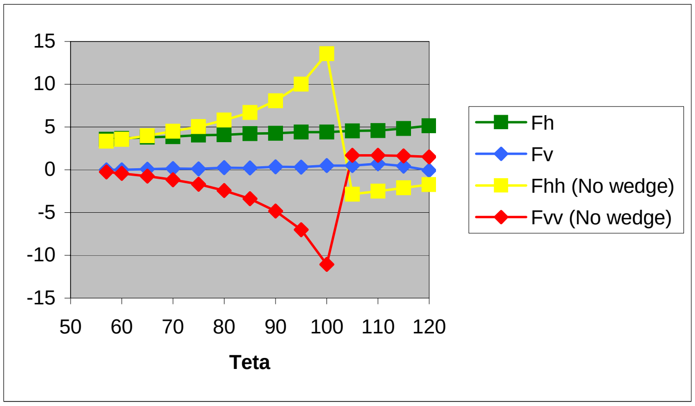

Figure 7: The cutting forces as a function of the blade angle, with and without a wedge.

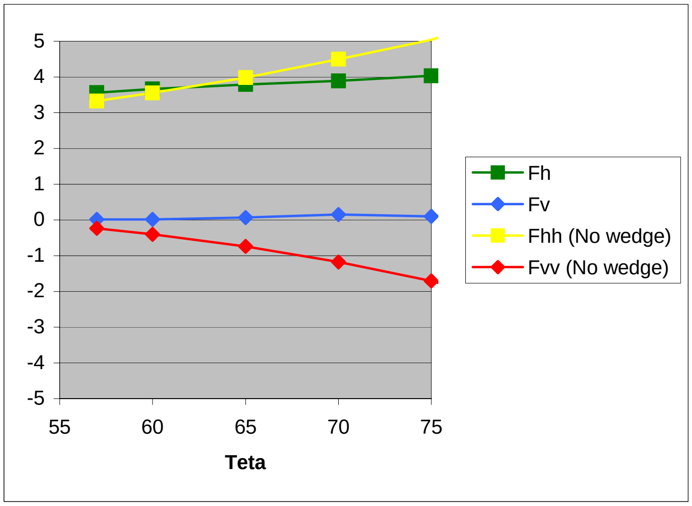

Figure 8: The cutting forces of Figure 7 enlarged.

So the solution is, using the equilibrium equations for the horizontal force, the vertical force and the moments on the wedge. The recipy to determine the cutting forces seems not to difficult now, but it requires a lot of calculations and understanding of the processes, because one also has to distinguish between the theory for small cutting angles and the wedge theory.

The following steps have to be taken to find the correct solution:

1. Determine the pore pressures p1, p2, p3, p4 using a finite element calculation or the method described by Miedema (2004), for a variety of shear angles β and wedge angles α around the expected solution.
2. Determine the shear angle β based on the equilibrium equations for the horizontal and vertical forces and the principle of minimum energy, whitch is equivalent to the minimum horizontal force. This also gives a value for the resulting force K_2 acting on the wedge.
3. Determine values of e2, e3 and e4 based on the results from the pore pressure calculations.
4. Determine the solutions of the equilibrium equations on the wedge and find the solution which has the minimum energy dissipation, resulting in the minimum horizontal force on the blade.
5. Determine the forces without a wedge with the theory for small cutting angles.
6. Determine which horizontal force is the smallest, with or without the wedge.

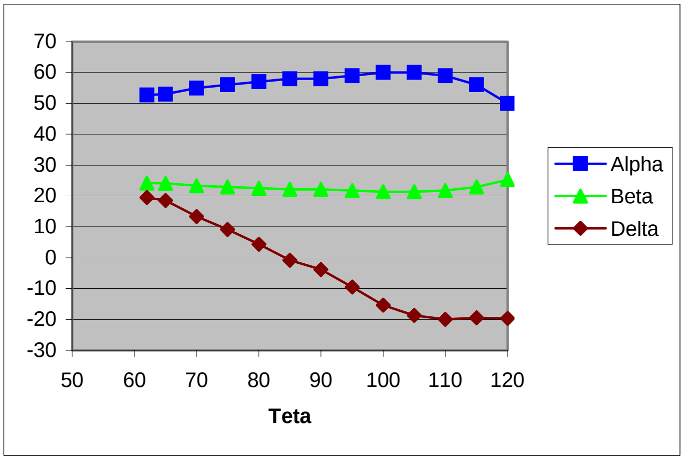

Figure 9: The wedge angle, shear angle and soil interface friction angle as a function of the blade angle.

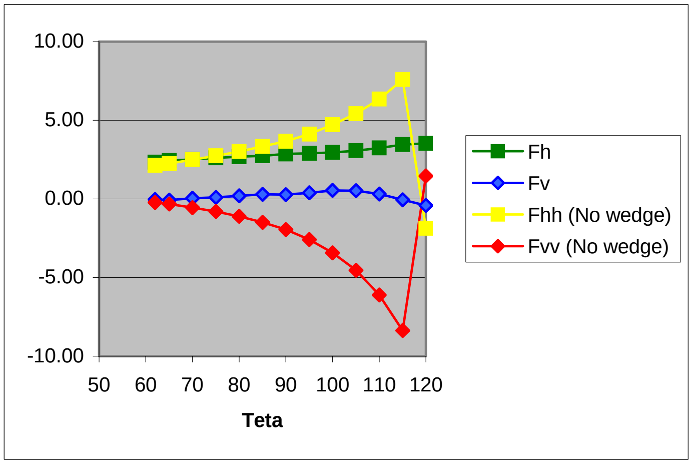

Figure 10: The cutting forces as a function of the blade angle, with and without a wedge.

CASE STUDIES

To illustrate the results of the calculation method, two cases will be discussed. In both cases calculations are carried out for blade angles of 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115 and 120 degrees, while the smallest angle is around 60 degrees depending on the possible solutions. Also in both cases the cutting forces are determined with and without a wedge, so its possible to carry out step 6.

The first case concerns a sand with an internal friction angle φ of 40°, a soil interface friction angle δ of 27° fully mobilized, a friction angle λ between the soil cut and the wedge equal to the internal friction angle, an initial permeability k_i of $6.2*10^{-5}$ m/s and a residual permeability k_{max} of $17*10^{-5}$ m/s. The blade dimensions are a width of 0.25 m and a height of 0.2 m, while a layer of sand of 0.05 m is cut with a cutting velocity of 0.3 m/s at a water depth of 0.6 m, matching the laboratory conditions. The values for the acting points of the forces, are e_2 = 0.35, e_3 = 0.55 and e_4 = 0.32, based on the finite element calculations carried out by Ma (2001). The second case concerns a sand with an internal friction angle of 30° and a soil interface friction angle of 20°, while the friction angle between the soil cut and the wedge equals the internal friction angle again. Since the tendencies of both cases are the same, both cases will be discussed simultaneously. Figures 6, 7 and 8 show the results for the 40° internal friction angle sand, while the Figures 9, 10 and 11 show the results for the 30° internal friction angle sand.

Figures 6 and 9 show the wedge angle, the shear angle and the mobilized soil interface friction angle as a function of the blade angle. The wedge angle found are about 90°-φ, which matches the theory of Hettiaratchi and Reece (1975). The shear angle is around 20° in both cases, but it is obvious that a larger internal friction angle gives a smaller shear angle β. The soil interface friction angle varies from minus the maximum mobilized friction to plus the maximum mobilized friction as is also shown in the force diagrams in Figure 5.

The Figures 7 and 10 show clearly how the cutting forces become infinite when the sum of the 4 angles involved is 180° and become negative when this sum is larger then 180°. The close up graphs 8 and 11 show that between 60° and 70° the cutting forces with the wedge become smaller then the cutting forces without the wedge. So the transition from the small angle theory to the wedge theory occurrs in this area, depending on the soil mechanical parameters and the geometry of the cutting process.

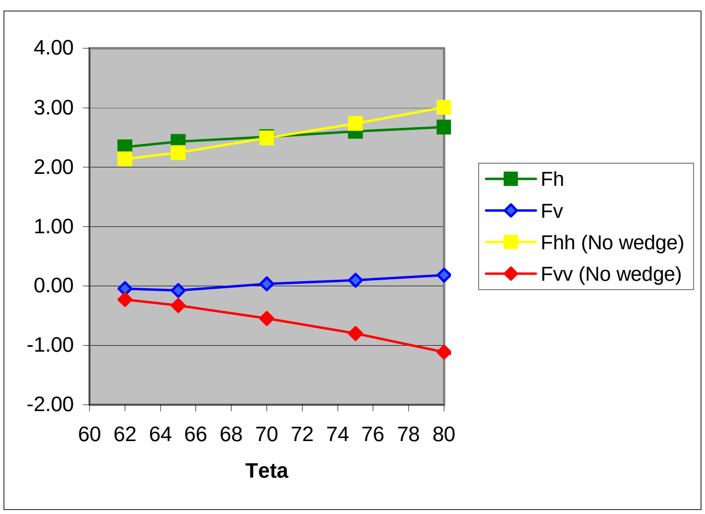

Figure 11: The cutting forces of Figure 10 enlarged.

CONCLUSIONS

The methodology applied gives satisfactory results to determine the cutting forces at large cutting angles. The results shown in this paper are valid for the non-cavitating cutting process and for the soils and geometry used in this paper.

The wedge angles found are, in general, a bit smaller then 90°-φ, so as a first approach this can be used. The mobilized soil interface friction angle varies from minus the maximum to plus the maximum depending on the blade angle.

The cutting forces with the wedge do not increase much when the cutting angle increases from 60° to 120°. If the ratio between the thickness of the layer cut and the blade height changes, also the values of the acting points e_2, e_3 and e_4 will change slightly.

It is not possible to find an explicit analytical solution for the wedge problem and its even difficult to automate the calculation method, since the solution depends strongly on the values of the acting points. More graphs will be published on http://www.dredgingengineering.com in the near future. For the cavitating cutting process, the phenomena are similar, but this will be described in a next paper.

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LIST OF SYMBOLS USED

b	width of the blade of blade element	m
e	volume strain	%
e_2, e_3, e_4	Acting point of cutting forces	-
F	cutting force (general)	kN
g	gravitation acceleration	m/s²
h_i	initial layer thickness	m
h_b	Blade height	m
k_i	initial permeability	m/s
k_{max}	maximum permeability	m/s
n_i	initial pore percentage	%
n_{max}	maximum pore percentage	%
$N_{1, 2, 3, 4}$	Normal force caused by grain stresses	kN
$p_{1, 2, 3, 4}$	average pore pressure	-
$S_{1, 2, 3, 4}$	Force caused by shear stresses	kN
v_c	cutting velocity perpendicular on the blade edge	m/s
$W_{1, 2, 3, 4}$	Force caused by pore pressures	kN
z	water depth	m
α	wedge angle(for wedge)	rad
β	shear angle	rad
θ	blade angle(for wedge)	rad
φ	angle of internal friction	rad
δ	soil/steel angle of friction	rad
ρ_w	water density	ton/m³
λ	angle of internal friction between wedge and layer cut	rad