DHLLDV Framework
Sliding Flow Transport

Dr.ir. Sape A. Miedema
Head of Studies
MSc Offshore & Dredging Engineering
&
Marine Technology
Associate Professor of
Dredging Engineering

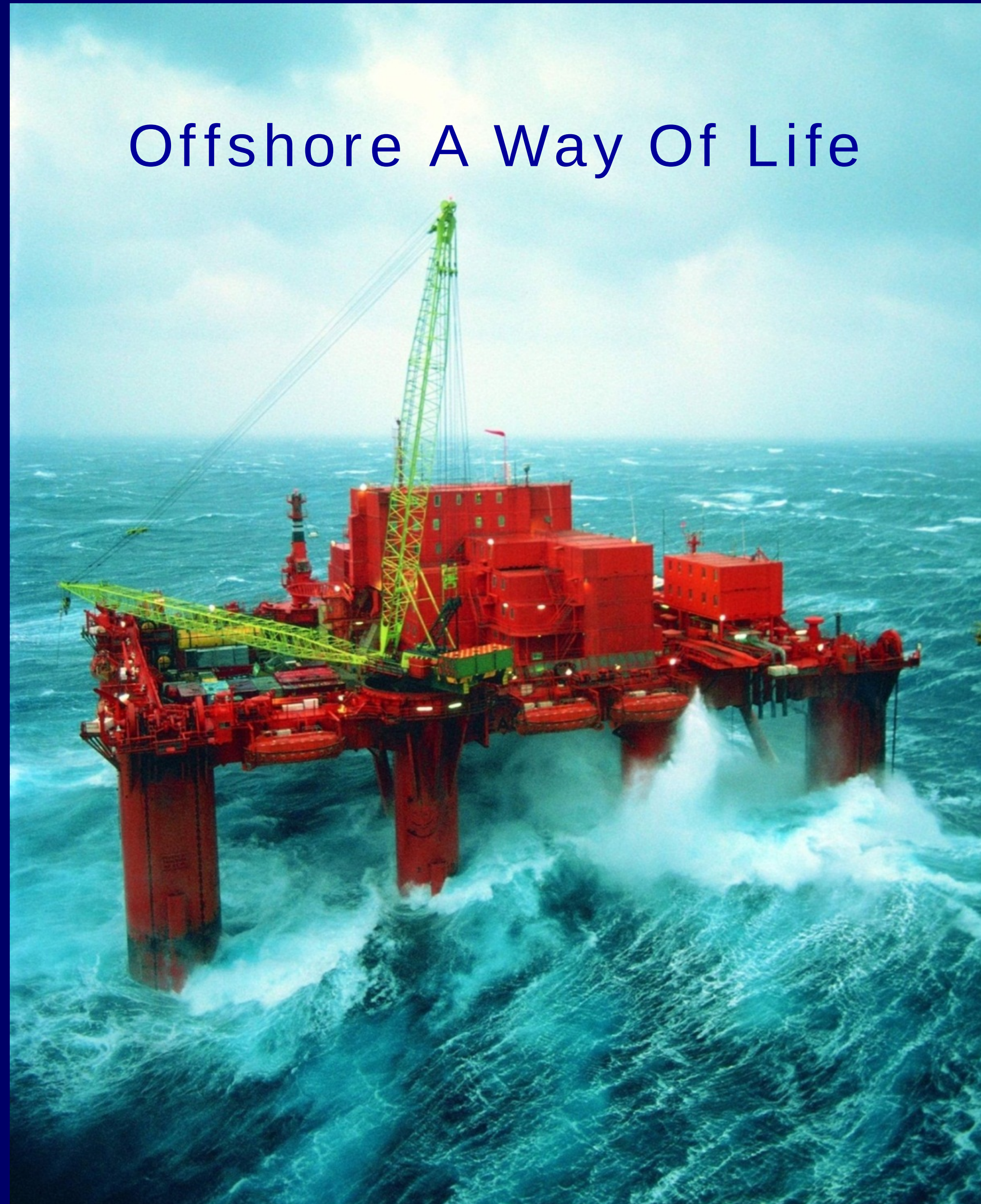
TU Delft
Delft University of Technology
Offshore & Dredging Engineering

TU Delft
Delft University of Technology
Offshore & Dredging Engineering

Dredging A Way Of Life

Delft University of Technology
Offshore & Dredging Engineering

Delft University of Technology – Offshore & Dredging Engineering



TU Delft
Delft University of Technology
Offshore & Dredging Engineering

Delft University of Technology – Offshore & Dredging Engineering

What is Offshore & Dredging Engineering?

Offshore & Dredging Engineering covers everything at sea that does not have the purpose of transporting goods & people and no fishery.

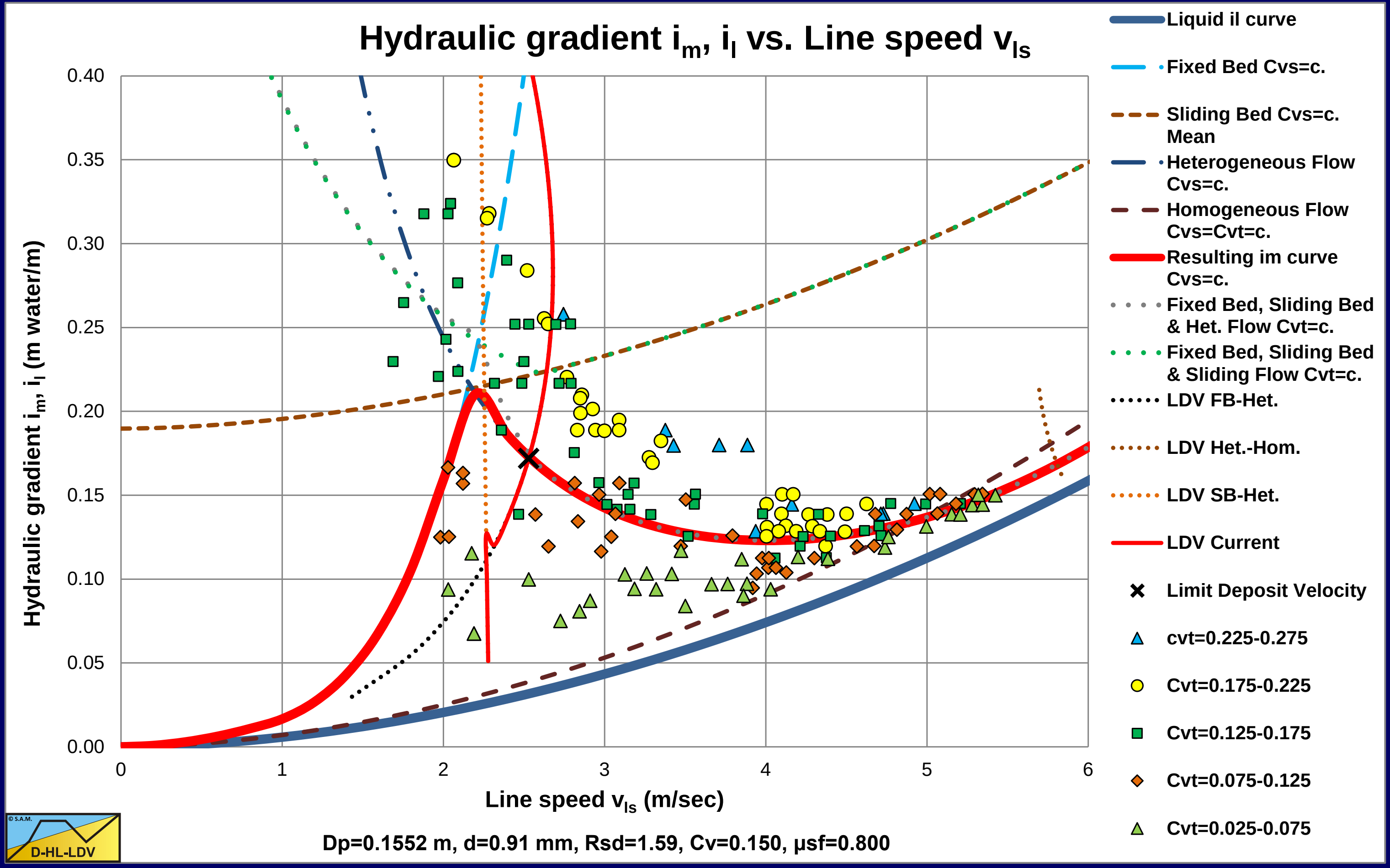
Research Question

In slurry transport several flow regimes are distinguished. The fixed or stationary bed regime, the sliding bed regime, the heterogeneous flow regime, the homogeneous flow regime and the sliding flow regime or fully stratified flow. These flow regimes each have their own physics. For the first 4 flow regimes many fundamental or empirical analytical models exist, however for the sliding flow regime or fully stratified flow there is only a simple criterion, the particle diameter should be larger than about 0.015 times the pipe diameter (Wilson & Sellgren). No good explanation is given for this criterion, nor a good definition is given.

Here a possible solution to this is given.

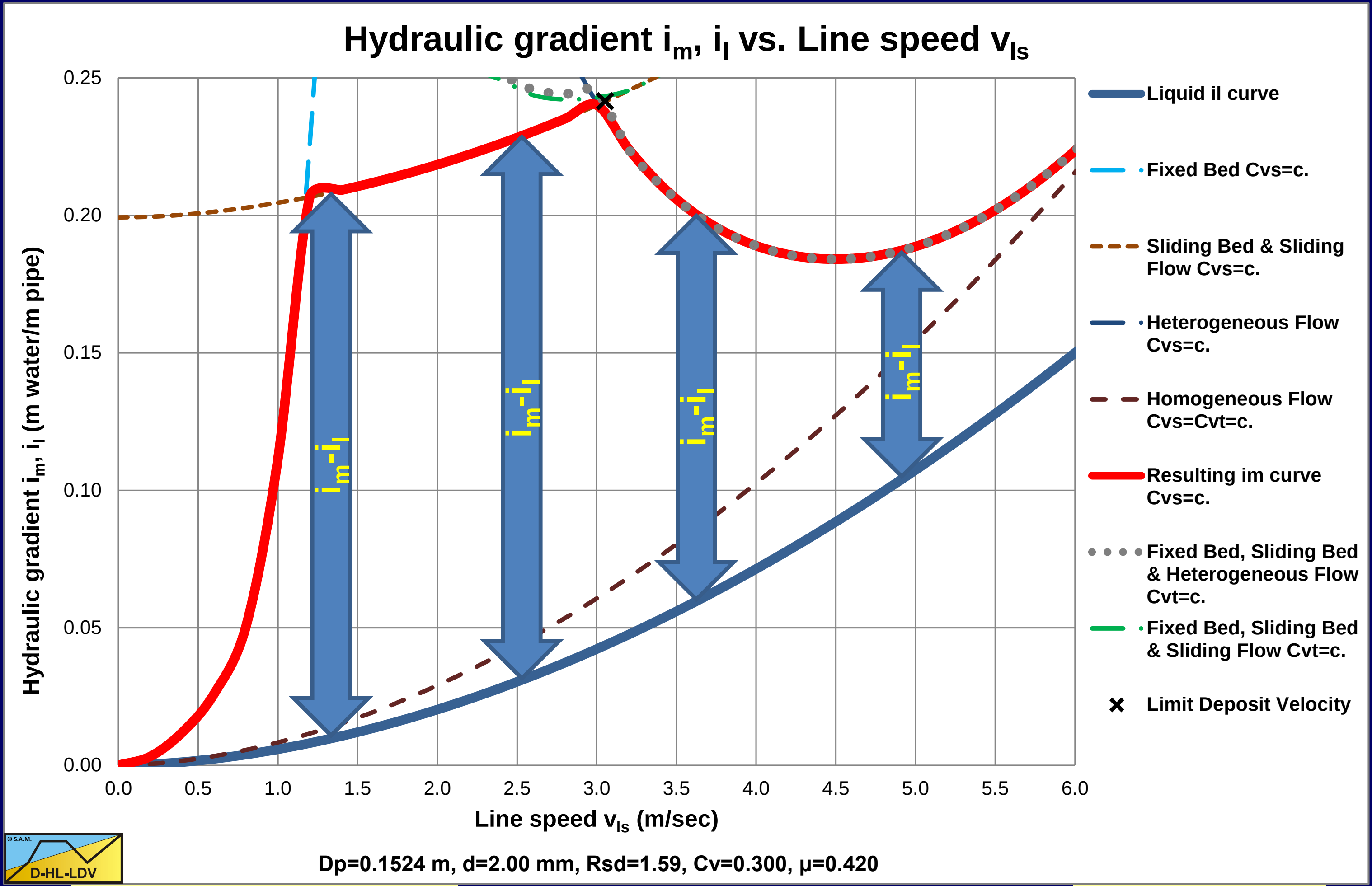
Introduction

Data from Yagi et al., C_{vs}



Delft University of Technology
Offshore & Dredging Engineering

DHLLDV Model, The Solids Effect



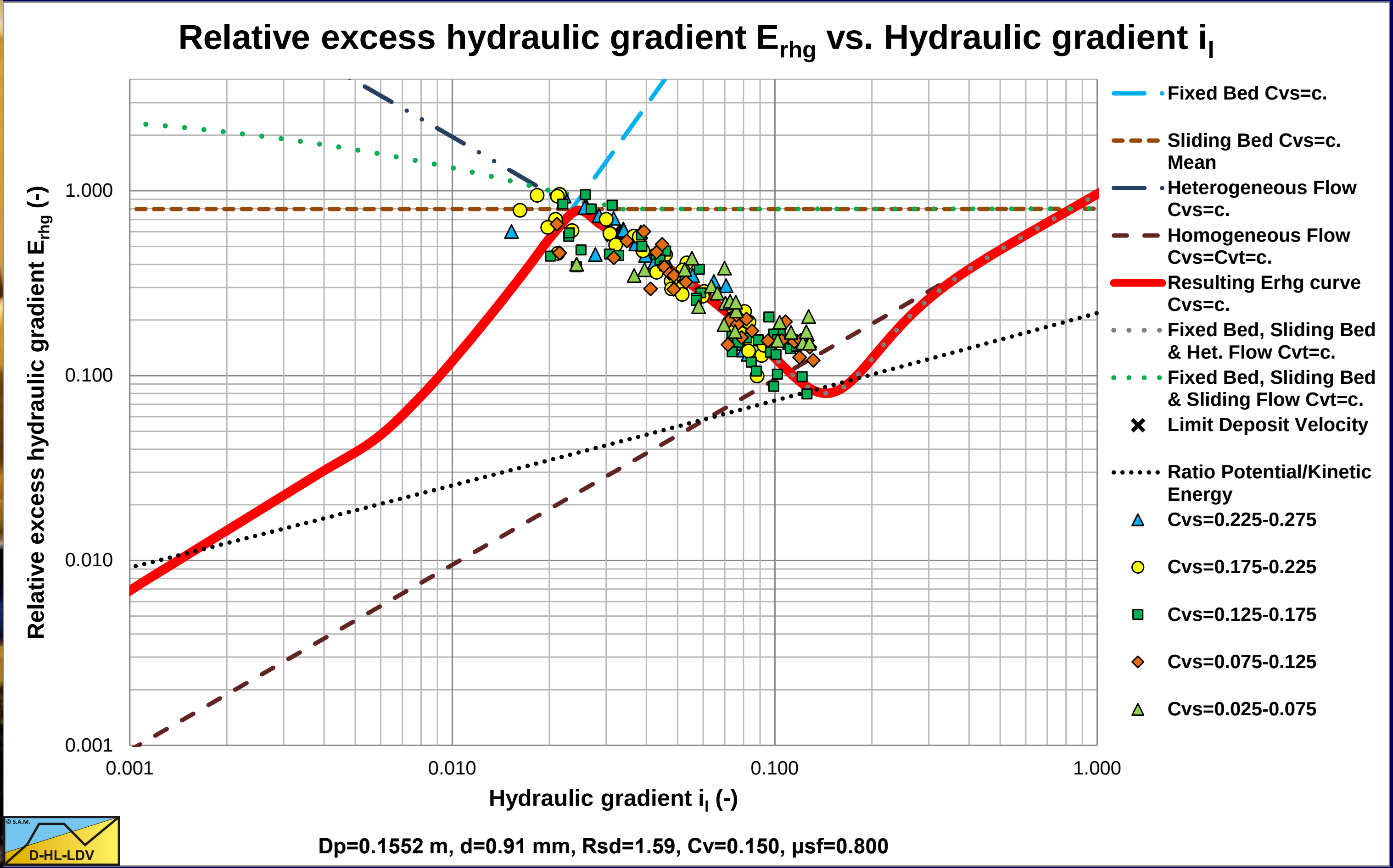
$$i_l = \frac{\Delta p_l}{\rho_l \cdot g \cdot \Delta L} = \frac{\lambda_l \cdot v_{ls}^2}{2 \cdot g \cdot D_p}$$

$$E_{rhg} = \frac{i_m - i_l}{R_{sd} \cdot C_v}$$

Delft University of Technology – Offshore & Dredging Engineering

TU Delft
Delft University of Technology
Offshore & Dredging Engineering

Data from Yagi et al., C_{vs}

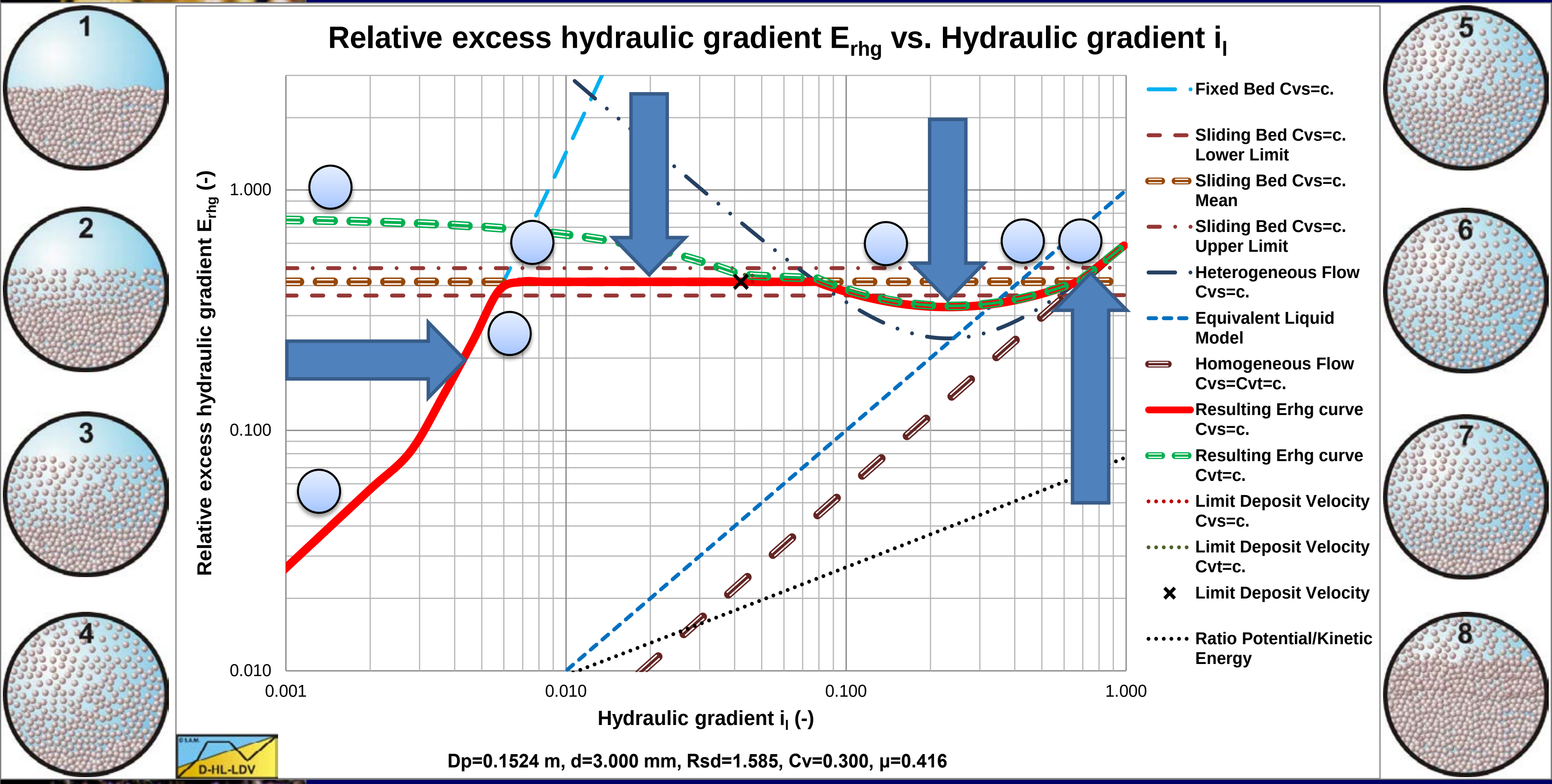


Sliding Flow Regime

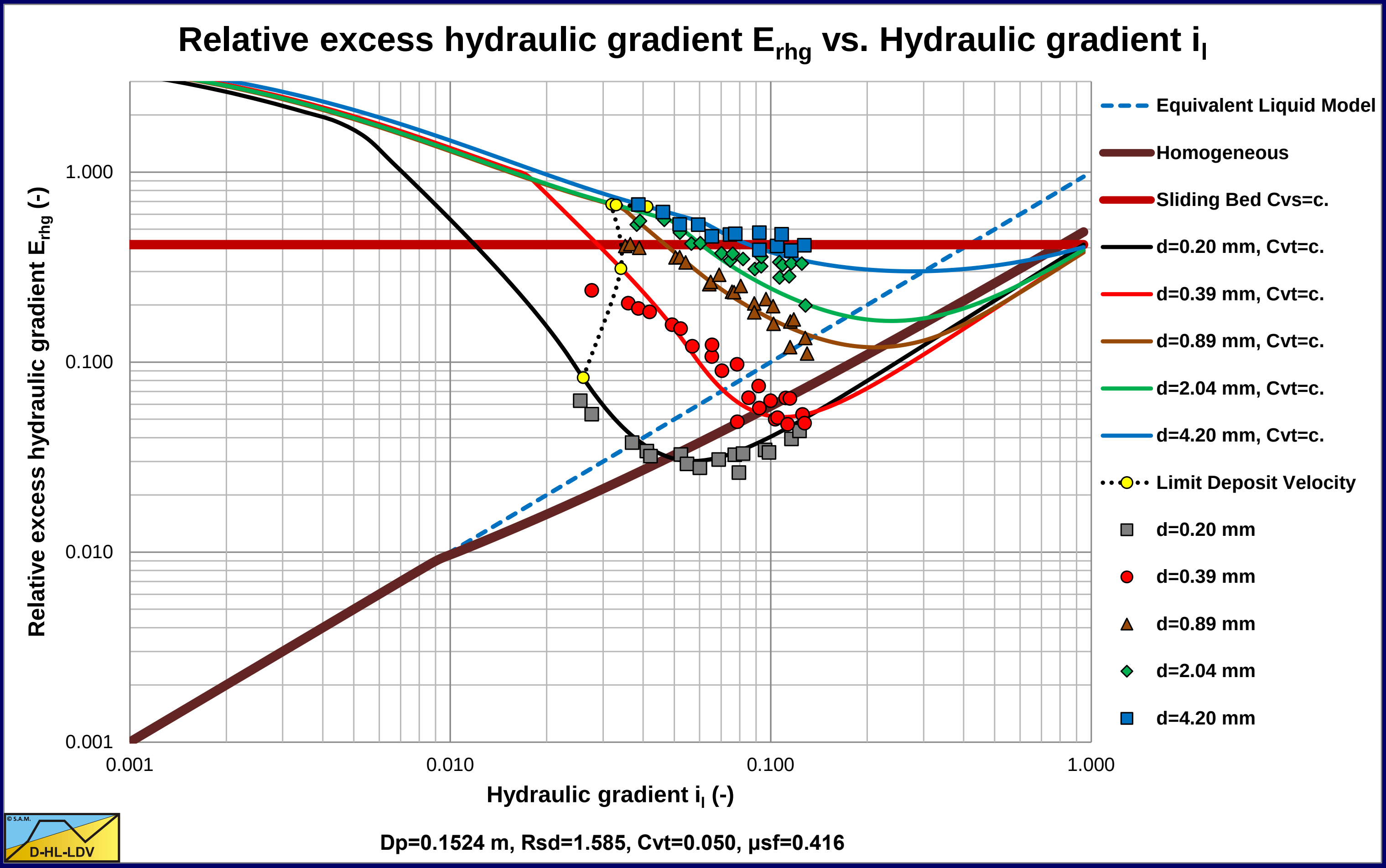
Chapter 7.7 & 8.8

Delft University of Technology
Offshore & Dredging Engineering

Flow Regimes

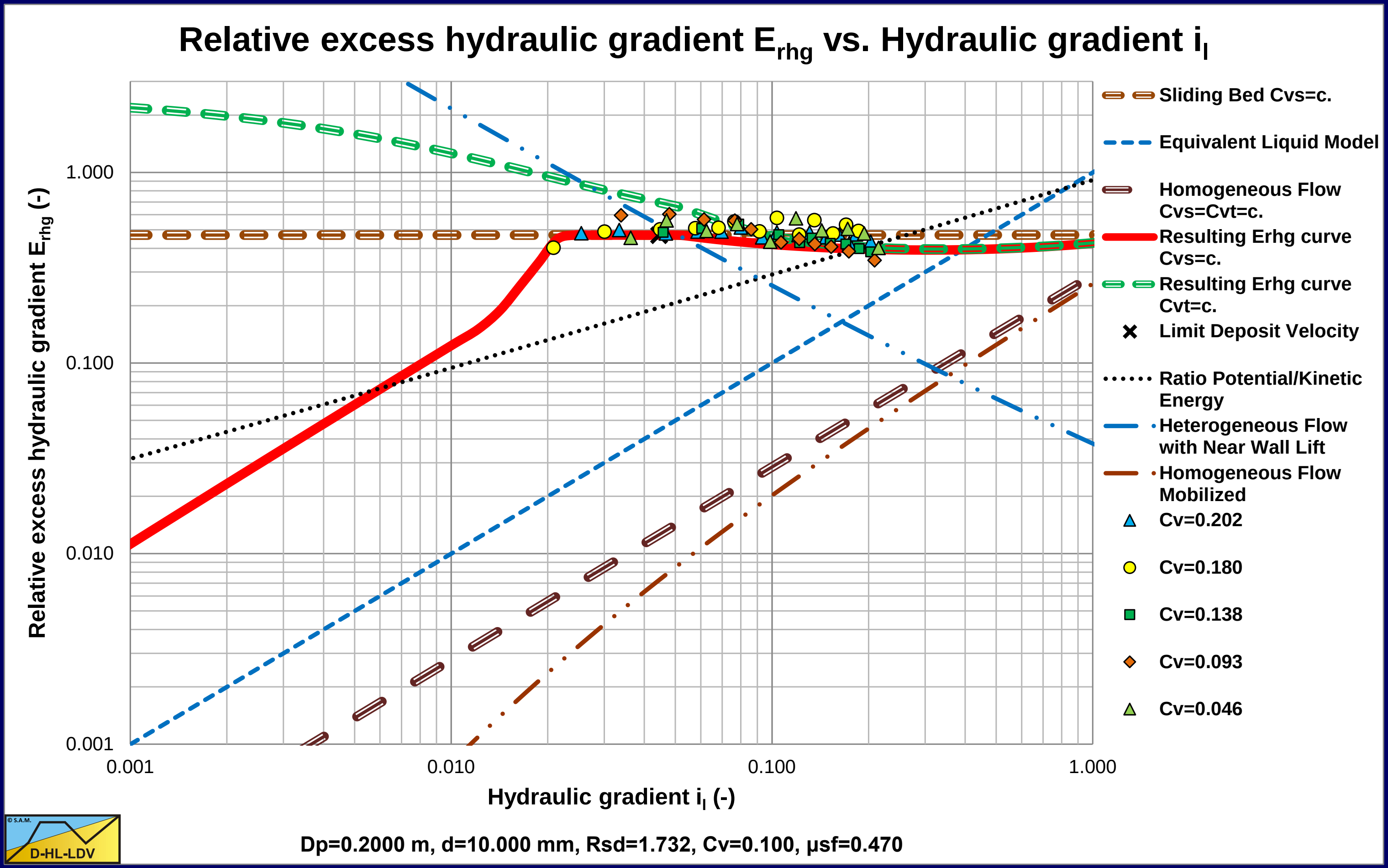


Verification & Validation, Durand



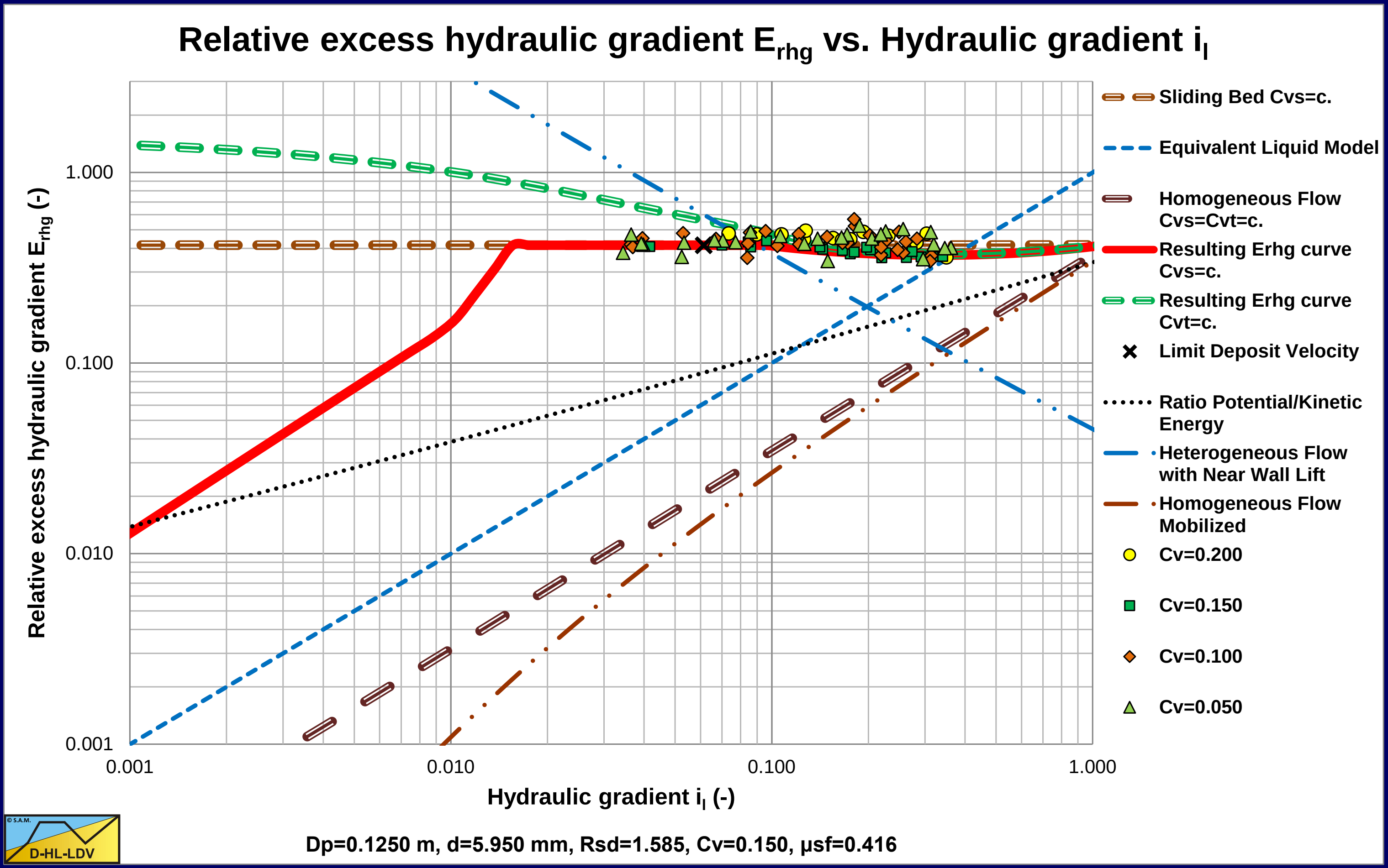
Durand, Condolios & Gibert (1952-1960)

Verification & Validation, Boothroyde



Boothroyde et al. (1979)

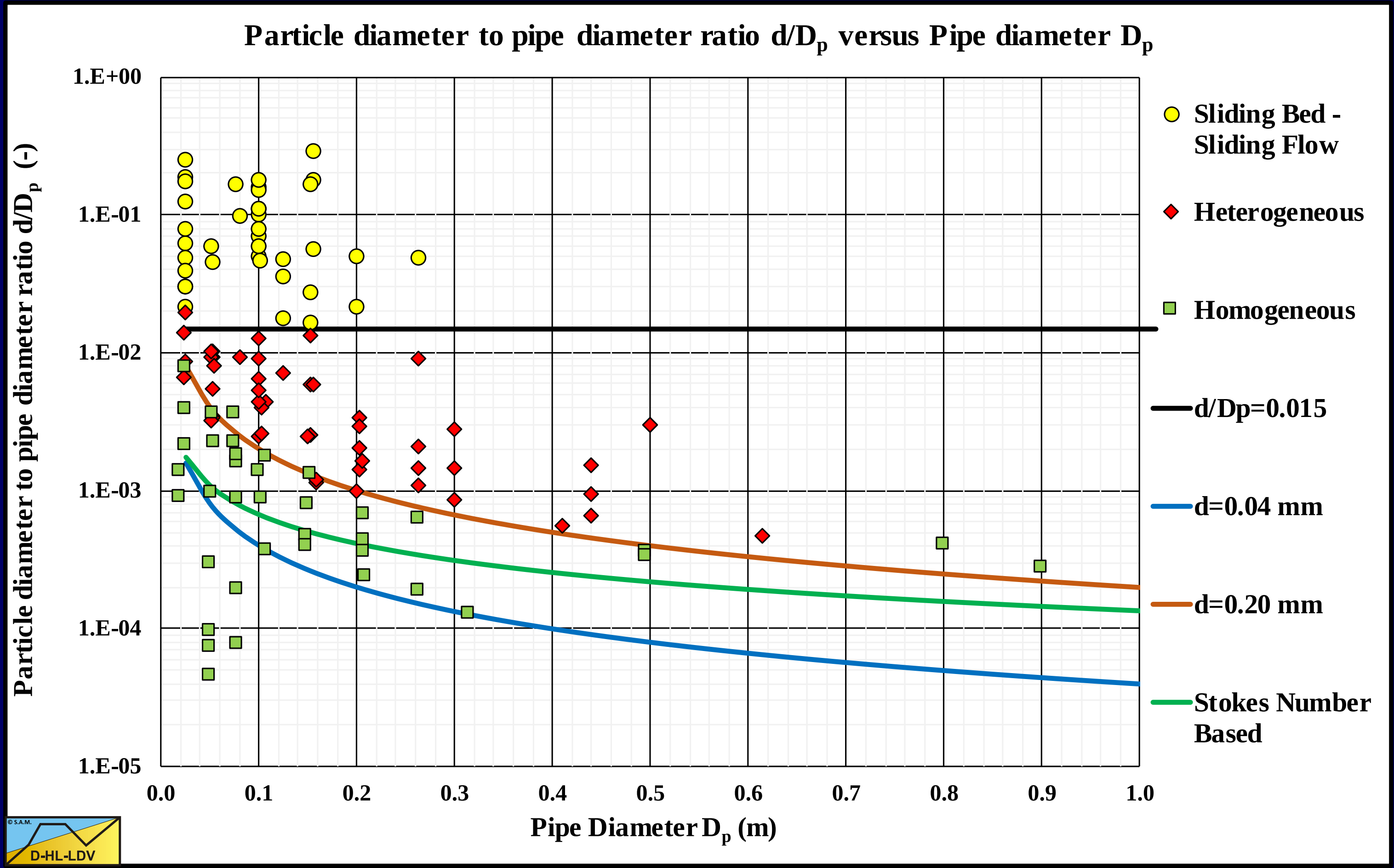
Verification & Validation, Wiedenroth



Wiedenroth (1967)

Delft University of Technology – Offshore & Dredging Engineering

Verification & Validation, All



Delft University of Technology – Offshore & Dredging Engineering

Phenomena

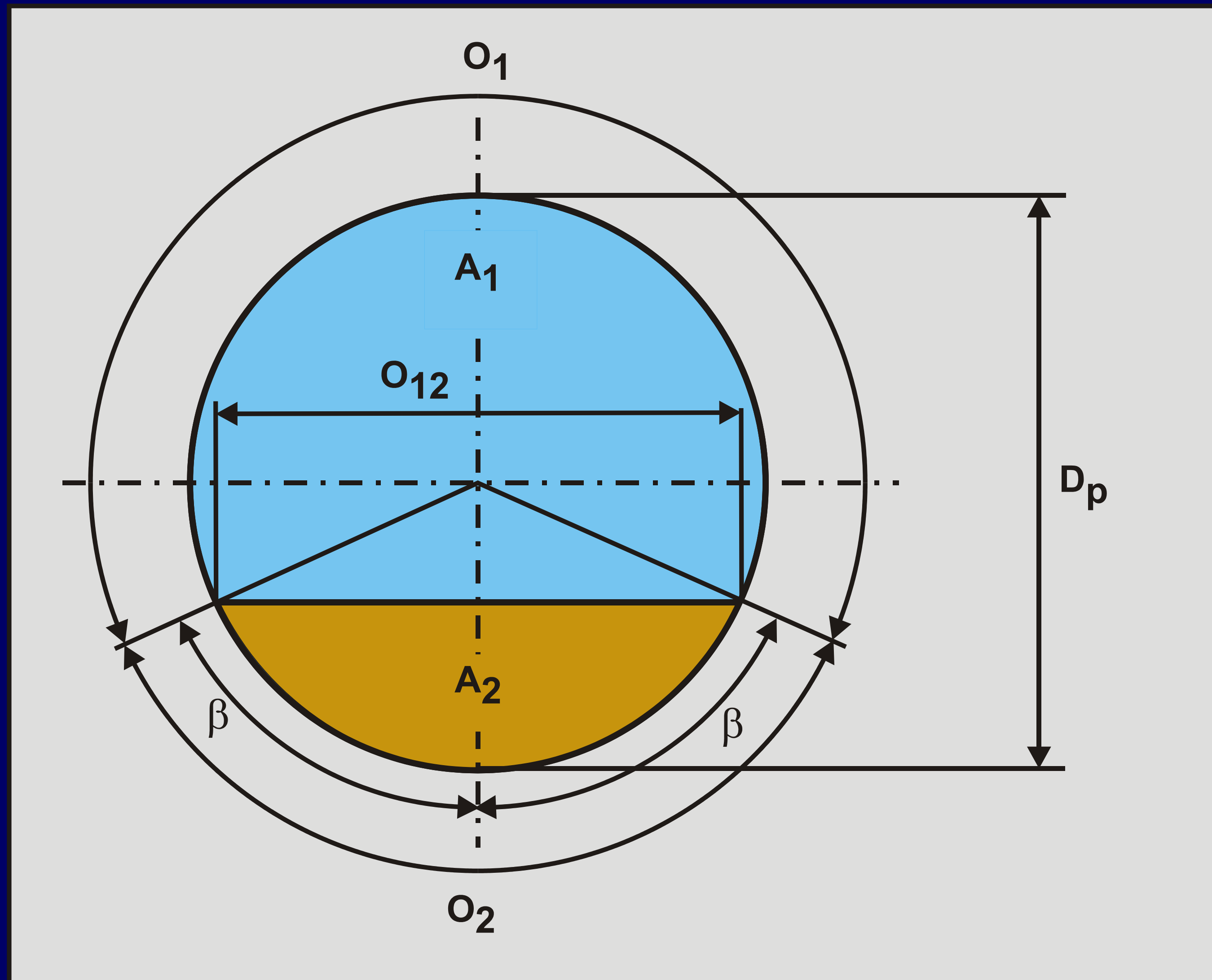
If the particle diameter to pipe diameter ratio is larger than about 0.015, the particles will not be suspended anymore, but stay in a fast flowing sort of bed, behaving according to a sliding bed.

For $0.0075 < d/D_p < 0.03$ there is a transition from heterogeneous behavior to sliding flow behavior.

Above $d/D_p = 0.03$ sliding flow is fully mobilized.

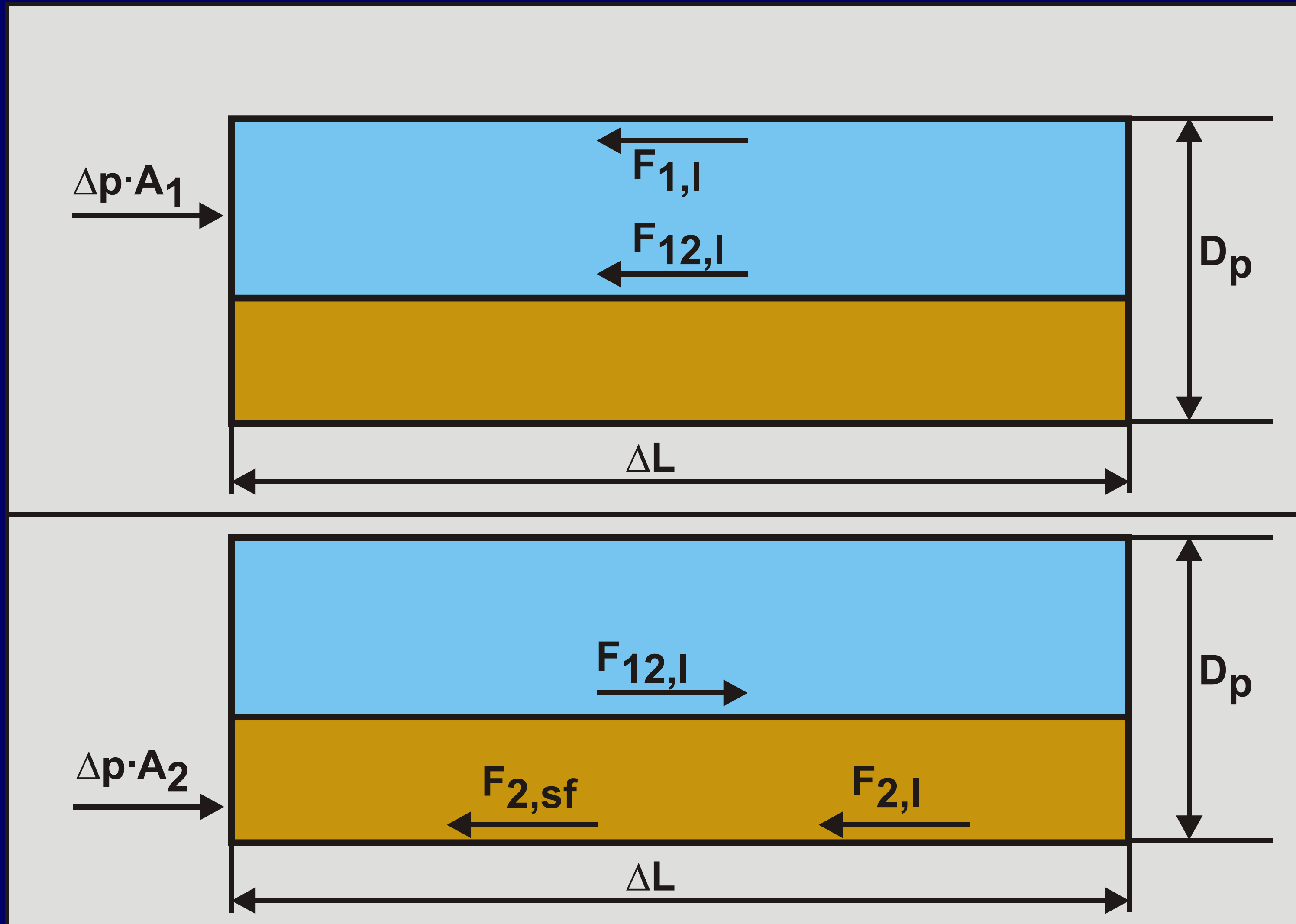
The higher the line speed the smaller the concentration of the flowing particles at the bottom of the pipe.

Definitions



Delft University of Technology
Offshore & Dredging Engineering

Equilibrium of Forces



Delft University of Technology
Offshore & Dredging Engineering

Derivation d/D_p Factor 1

Deposition = Suspension

Criterion: $v_t = u_*(\text{bed})$

Equilibrium of forces on layer above the sliding bed

$$F_{12,l} + F_{1,l} = \Delta p \cdot A_1$$

$$\rho_l \cdot u_*^2 \cdot D_p \cdot \sin(\beta) \cdot \Delta L + \alpha_\tau \cdot \rho_l \cdot u_*^2 \cdot D_p \cdot (\pi - \beta) \cdot \Delta L$$

$$= \Delta p \cdot A_p \cdot (1 - C_{vr})$$

$$\Delta p = \frac{\rho_l \cdot u_*^2 \cdot D_p \cdot \Delta L \cdot \left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta)\right)}{A_p \cdot (1 - C_{vr})}$$

Derivation d/D_p Factor 2

Equilibrium of forces the bed

$$F_{12,l} + \Delta p \cdot A_2 = F_{2,sf} + F_{2,l} \qquad F_{2,l} = \text{small and is neglected here}$$

$$\rho_l \cdot u_*^2 \cdot D_p \cdot \sin(\beta) \cdot \Delta L + \Delta p \cdot A_p \cdot C_{vr} = \mu_{sf} \cdot (\rho_s - \rho_l) \cdot g \cdot A_p \cdot C_{vs} \cdot \Delta L$$

$$\rho_l \cdot u_*^2 \cdot D_p \cdot \sin(\beta) + \frac{\rho_l \cdot u_*^2 \cdot D_p \cdot \left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta)\right)}{(1 - C_{vr})} \cdot C_{vr}$$

$$= \mu_{sf} \cdot (\rho_s - \rho_l) \cdot g \cdot A_p \cdot C_{vs}$$

$$\frac{\rho_l \cdot u_*^2 \cdot D_p \cdot \left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta) \cdot C_{vr}\right)}{(1 - C_{vr})} = \mu_{sf} \cdot (\rho_s - \rho_l) \cdot g \cdot A_p \cdot C_{vs}$$

Derivation d/D_p Factor 3

Equilibrium of forces the bed

$$\frac{\rho_l \cdot u_*^2 \cdot D_p \cdot \left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta) \cdot C_{vr}\right)}{\left(1 - C_{vr}\right)} = \mu_{sf} \cdot \left(\rho_s - \rho_l\right) \cdot g \cdot A_p \cdot C_{vs}$$

$$u_*^2 = \frac{\mu_{sf} \cdot \left(\rho_s - \rho_l\right) \cdot g \cdot A_p \cdot C_{vb} \cdot C_{vr} \cdot \left(1 - C_{vr}\right)}{\rho_l \cdot D_p \cdot \left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta) \cdot C_{vr}\right)}$$

$$u_*^2 = \frac{\pi}{4} \cdot \frac{\mu_{sf} \cdot R_{sd} \cdot g \cdot D_p \cdot C_{vb} \cdot C_{vr} \cdot \left(1 - C_{vr}\right)}{\left(\sin(\beta) + \alpha_\tau \cdot (\pi - \beta) \cdot C_{vr}\right)}$$

Derivation d/D_p Factor 4

Friction velocity squared

$$u_*^2 = \frac{\pi}{4} \cdot \frac{\mu_{sf} \cdot R_{sd} \cdot g \cdot D_p \cdot C_{vb} \cdot C_{vr} \cdot \left(1 - C_{vr}\right)}{\left(\sin\left(\beta\right) + \alpha_\tau \cdot \left(\pi - \beta\right) \cdot C_{vr}\right)}$$

Settling velocity

$$v_t = \sqrt{\frac{4}{3} \cdot \frac{R_{sd} \cdot g \cdot d \cdot \psi}{C_D}} \quad \Rightarrow \quad v_t^2 = \frac{4}{3} \cdot \frac{R_{sd} \cdot g \cdot d \cdot \psi}{C_D}$$

$$\Rightarrow$$

$$\frac{4}{3} \cdot \frac{R_{sd} \cdot g \cdot d \cdot \psi}{C_D} = \frac{\pi}{4} \cdot \frac{\mu_{sf} \cdot R_{sd} \cdot g \cdot D_p \cdot C_{vb} \cdot C_{vr} \cdot \left(1 - C_{vr}\right)}{\left(\sin\left(\beta\right) + \alpha_\tau \cdot \left(\pi - \beta\right) \cdot C_{vr}\right)}$$

TU Delft
Delft University of Technology
Offshore & Dredging Engineering

Derivation d/D_p Factor 5

With wall shear stress

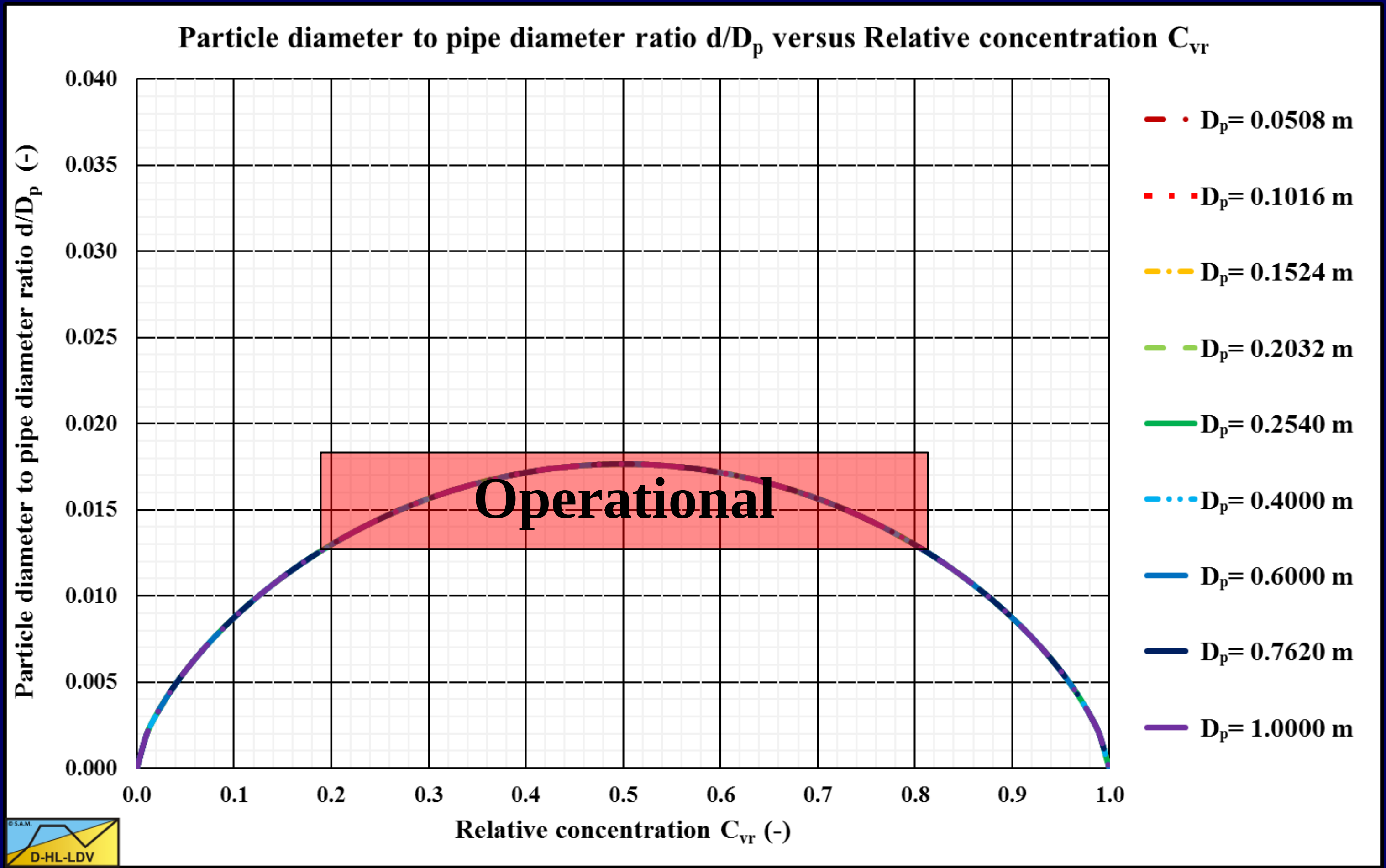
$$\frac{d}{D_p}=\frac{3\cdot\pi}{16}\cdot\frac{C_D}{\psi}\cdot\frac{\mu_{sf}\cdot C_{vb}\cdot C_{vr}\cdot\left(1-C_{vr}\right)}{\left(\sin\left(\beta\right)+\alpha_{\tau}\cdot\left(\pi-\beta\right)\cdot C_{vr}\right)}$$

Without wall shear stress

$$\frac{d}{D_p}=\frac{3\cdot\pi}{16}\cdot\frac{C_D}{\psi}\cdot\frac{\mu_{sf}\cdot C_{vb}\cdot C_{vr}\cdot\left(1-C_{vr}\right)}{\sin\left(\beta\right)}$$

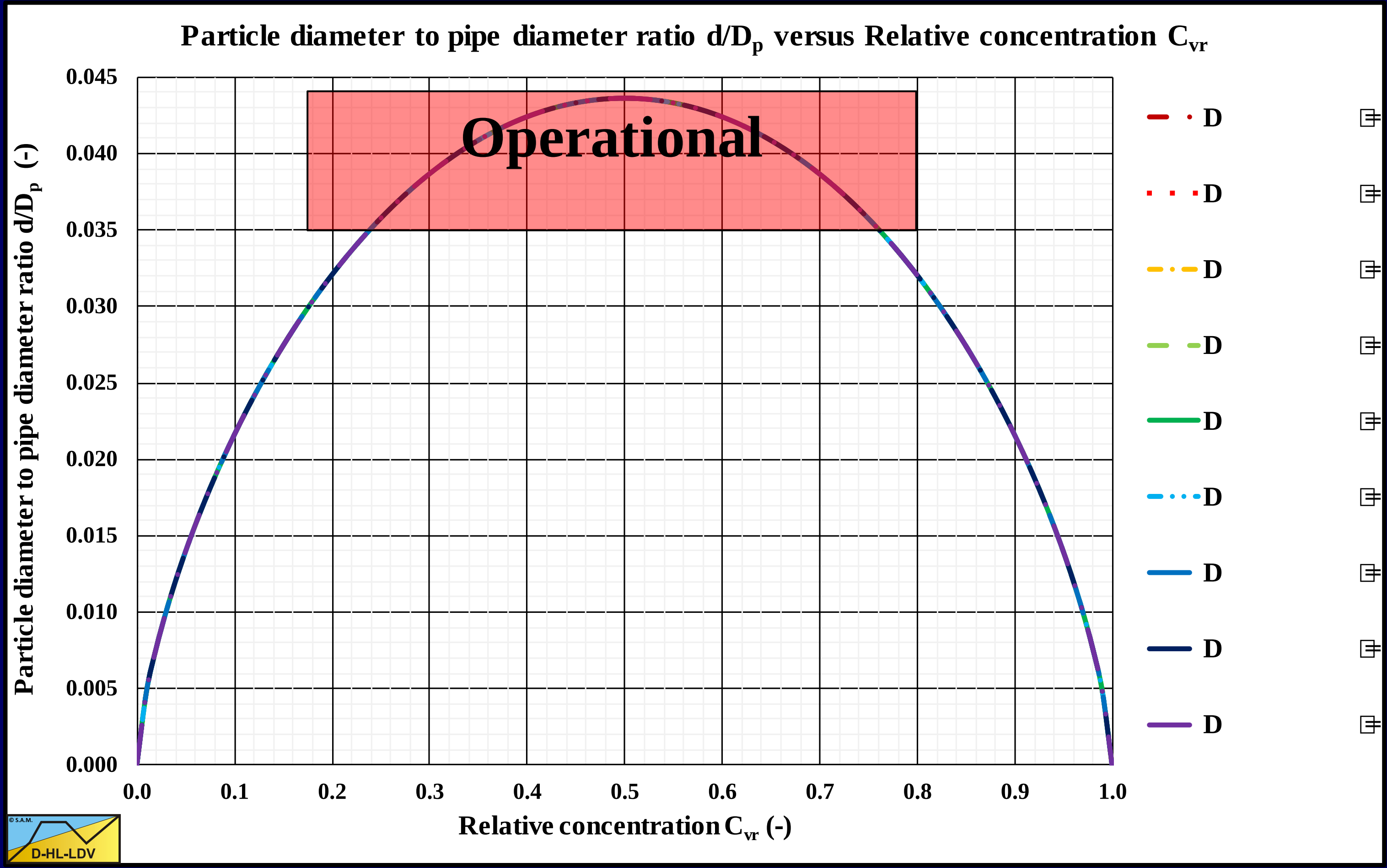
Delft University of Technology
Offshore & Dredging Engineering

Particle to Pipe Diameter Ratio, Spheres



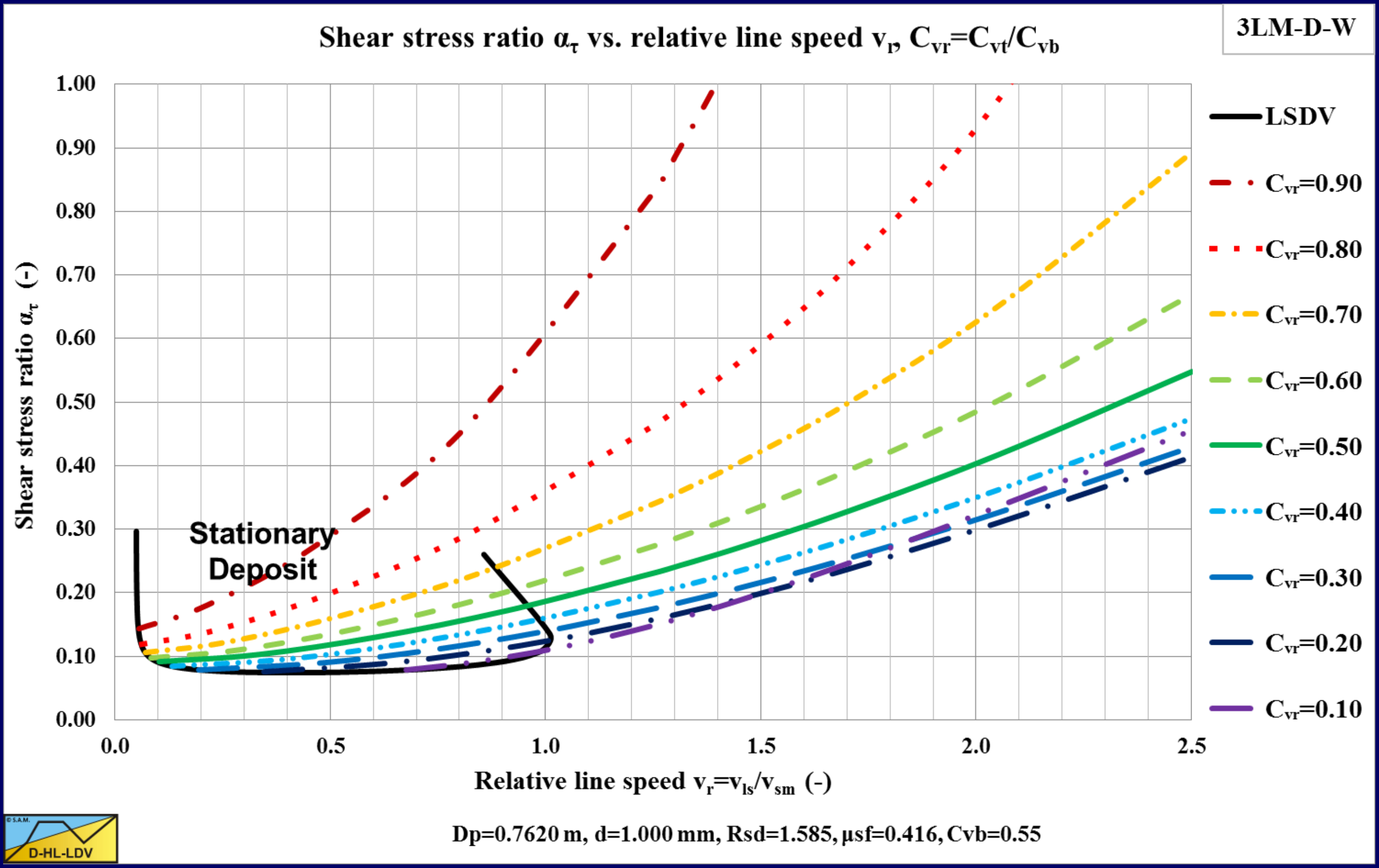
d/D_p ratio for spheres without τ_1, matching Wilson & Sellgren (0.013-0.018).

Particle to Pipe Diameter Ratio, Gravels

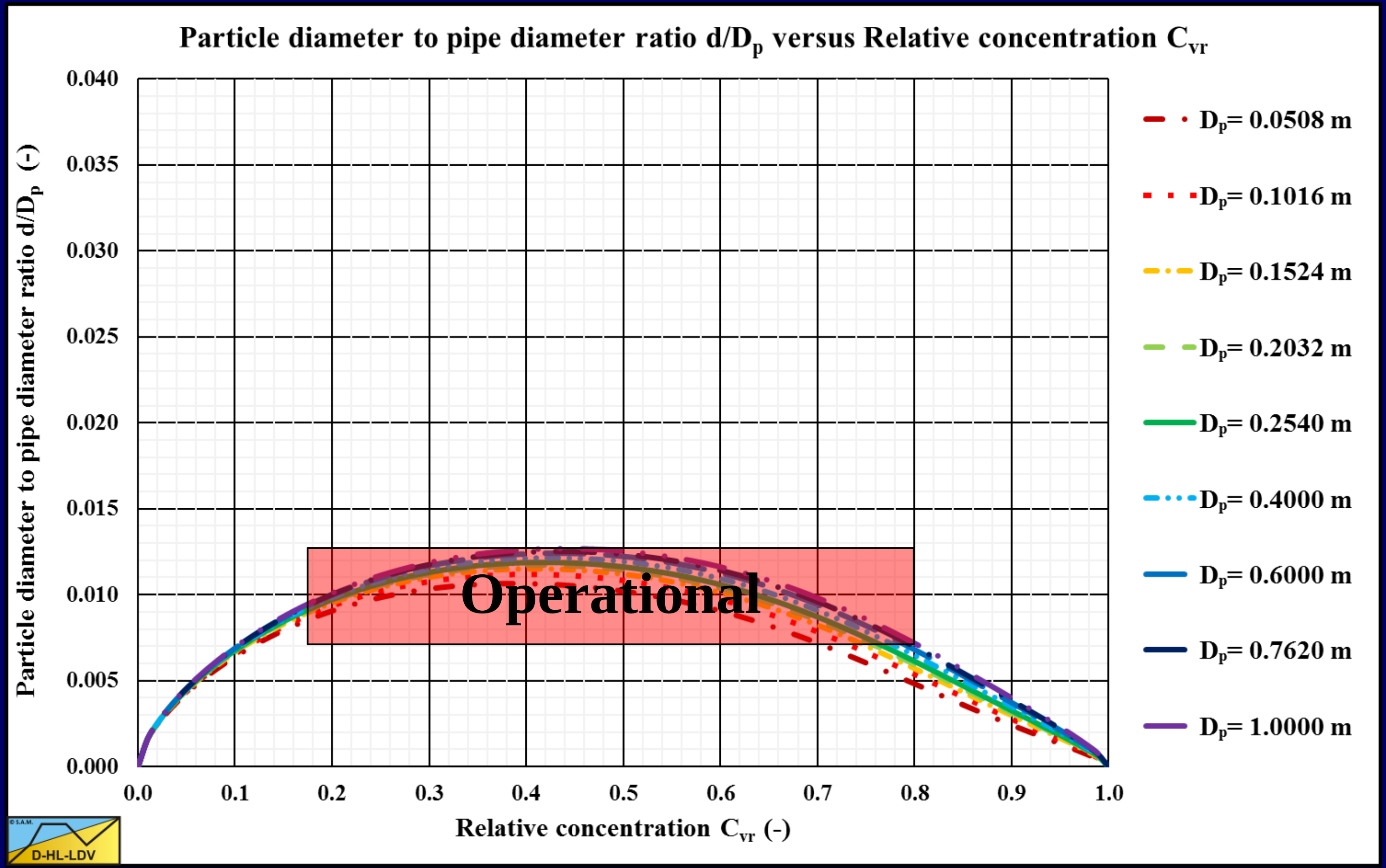


d/D_p ratio for gravels without τ_1, higher than Wilson & Sellgren (0.013-0.018).

Shear Stress Ratio

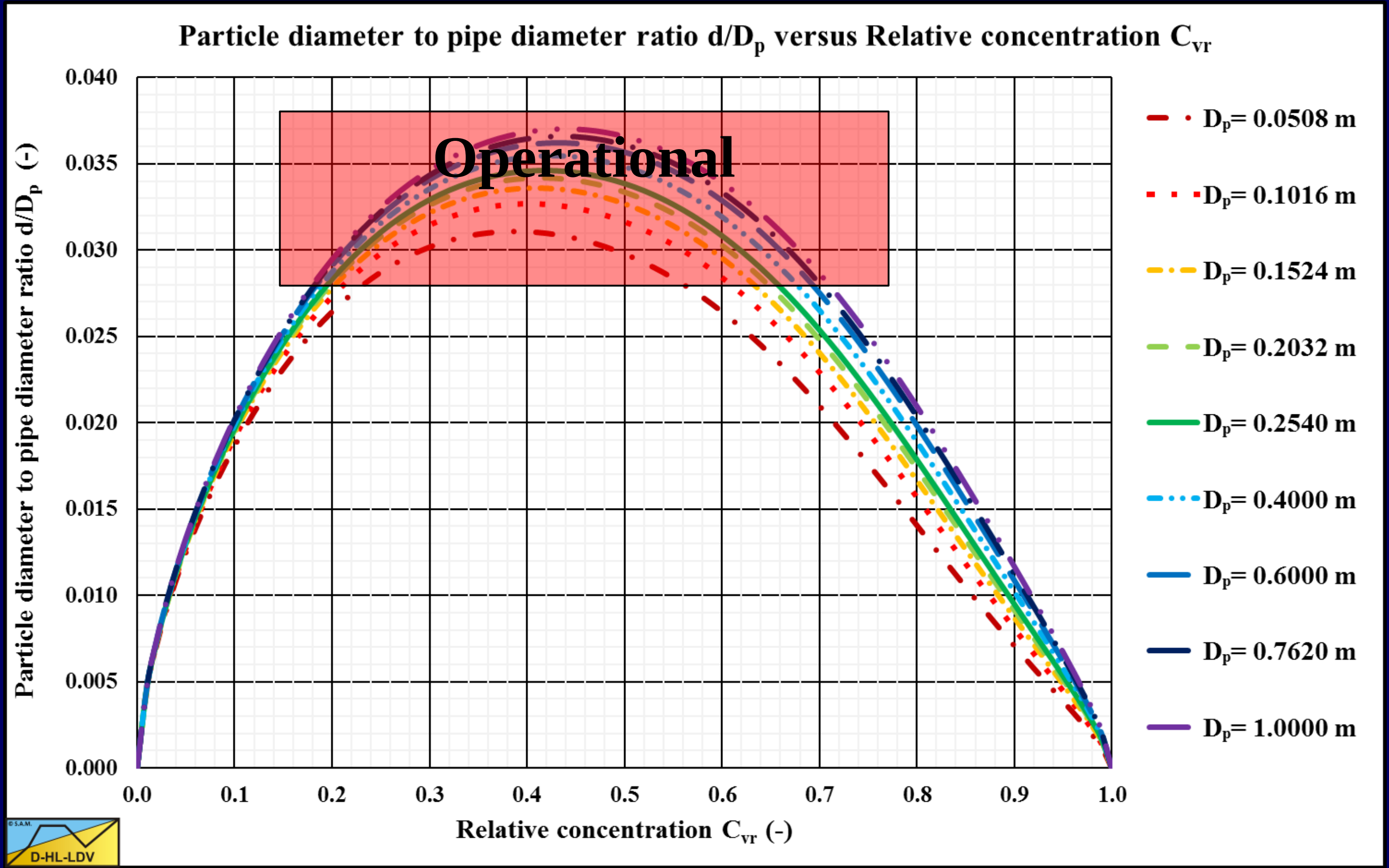


Particle to Pipe Diameter Ratio, Spheres



d/D_p ratio for spheres with τ_1, still matching Wilson & Sellgren (0.013-0.018).

Particle to Pipe Diameter Ratio, Gravels



d/D_p ratio for gravels with τ_1, higher than Wilson & Sellgren (0.013-0.018).

Conclusions

- It is possible to derive a more fundamental equation for the transition between the heterogeneous flow regime and the sliding flow regime based on the assumption deposition=suspension.
- This fundamental equation matches the d/D_p ratio of 0.015 closely for spheres.
- For sand and gravel particles the d/D_p ratio is closer to 0.03.
- The d/D_p ratio depends weakly on the pipe diameter and the particle diameter.
- The d/D_p ratio depends strongly on the particle shape and the particle drag coefficient.

Questions?