DHLLDV Framework The Concentration Distribution

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Dredging A Way Of Life

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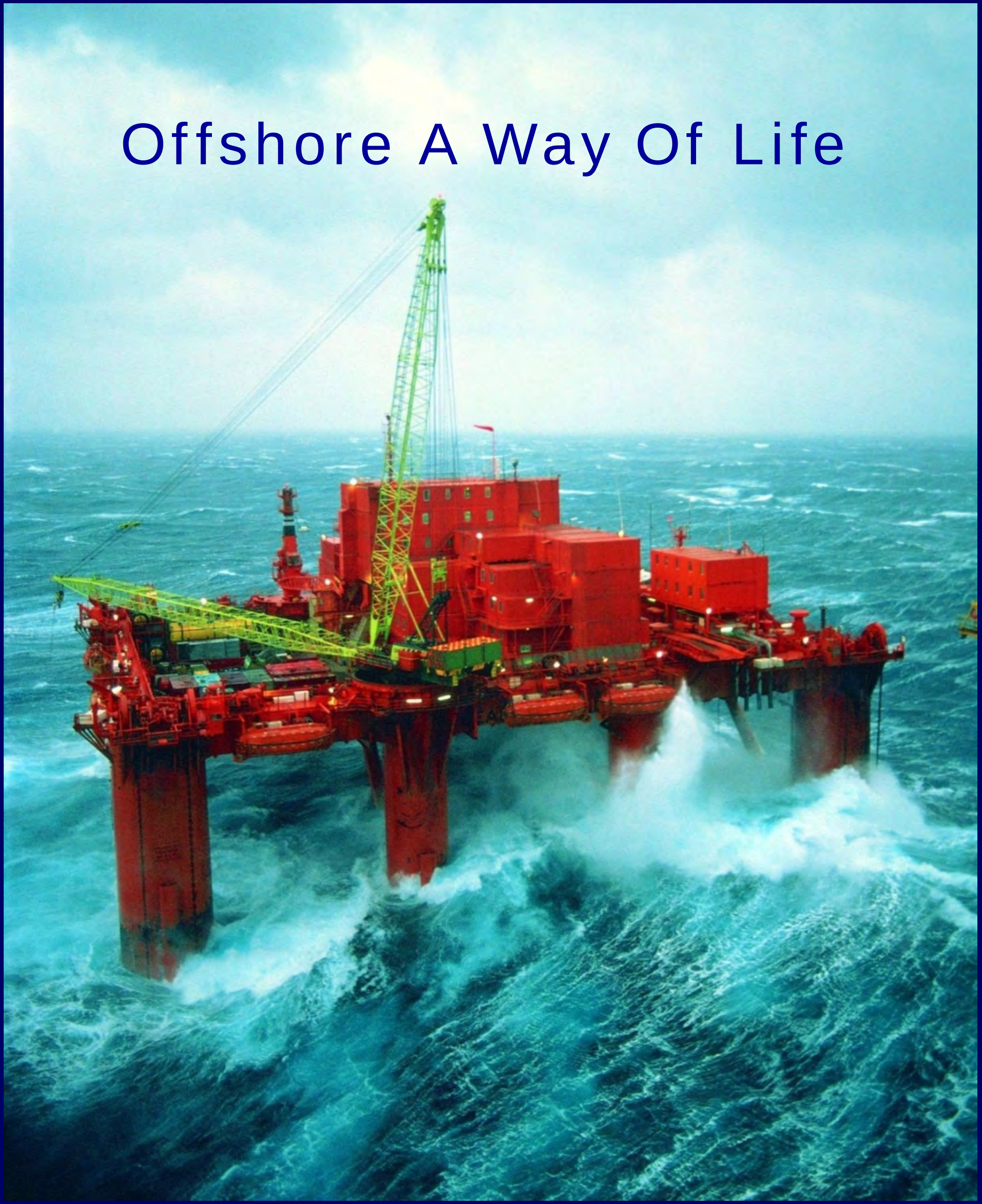
Offshore A Way Of Life



What is Offshore & Dredging Engineering?

Offshore & Dredging Engineering covers everything at sea that does not have the purpose of transporting goods & people and no fishery.

Research Question

Problem definition:

Existing methods for determining the concentration distribution in slurry transport are based on an average hindered settling velocity according to Richardson & Zaki for 2D channel flow.

- In pipe flow the flow is 3D.
- The hindered settling depends on the local concentration and thus on the position in the pipe.
- The Richardson & Zaki equation is valid for low concentrations.
- The advection diffusion equation is valid for small particles.

Concentration Distribution

Chapter 7.10 & 8.13

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Advection-Diffusion Equation

$$C_{vs}(r) \cdot v_{th} + \varepsilon_s \cdot \frac{dC_{vs}(r)}{dr} = C_{vs}(r) \cdot v_{th} + \beta_{sm} \cdot \varepsilon_m \cdot \frac{dC_{vs}(r)}{dr} = 0$$

$$\frac{dC_{vs}(r)}{C_{vs}(r)} = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot dr \quad \Rightarrow \quad \ln\left(C_{vs}(r)\right) = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot r + C$$

$$C_{vs}(r) = C_{vB} \cdot e^{-\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot r} = C_{vB} \cdot e^{-12 \cdot \frac{v_{th}}{\beta_{sm} \cdot \kappa \cdot u_*} \cdot \frac{r}{D_p}}$$

- Used in the Wasp model.
- Derived for 2D open channel flow.
- It is assumed particles follow the turbulent eddies.
- It is assumed the (hindered) terminal settling velocity is constant over the cross section of the pipe.
- There is no influence of the pipe wall.

Diffusivity Based on LDV

$$C_{vB} = C_{vs} \cdot \frac{\left(\dfrac{12 \cdot v_{th}}{\beta_{sm} \cdot \kappa \cdot u_*}\right)}{\left(1 - e^{-12 \cdot \frac{v_{th}}{\beta_{sm} \cdot \kappa \cdot u_*}}\right)} \quad \Rightarrow \quad C_{vb} = C_{vs} \cdot \frac{\left(\dfrac{12 \cdot v_{th,ldv}}{\beta_{sm,ldv} \cdot \kappa \cdot u_{*,ldv}}\right)}{\left(1 - e^{-12 \cdot \frac{v_{th,ldv}}{\beta_{sm} \cdot \kappa \cdot u_{*,ldv}}}\right)}$$

$$\beta_{sm,ldv} = 12 \cdot \frac{C_{vs}}{C_{vb}} \cdot \frac{v_{th,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}} = 12 \cdot C_{vr} \cdot \frac{v_{th,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}}$$

$$C_{vs}(r) = C_{vB} \cdot e^{-12 \cdot \frac{v_{th}}{\left(12 \cdot C_{vr} \cdot \frac{v_{th,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}}\right) \cdot \kappa \cdot u_*} \cdot \frac{r}{D_p}} = C_{vB} \cdot e^{-\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}} \cdot \frac{r}{D_p}}$$

The Bottom Concentration

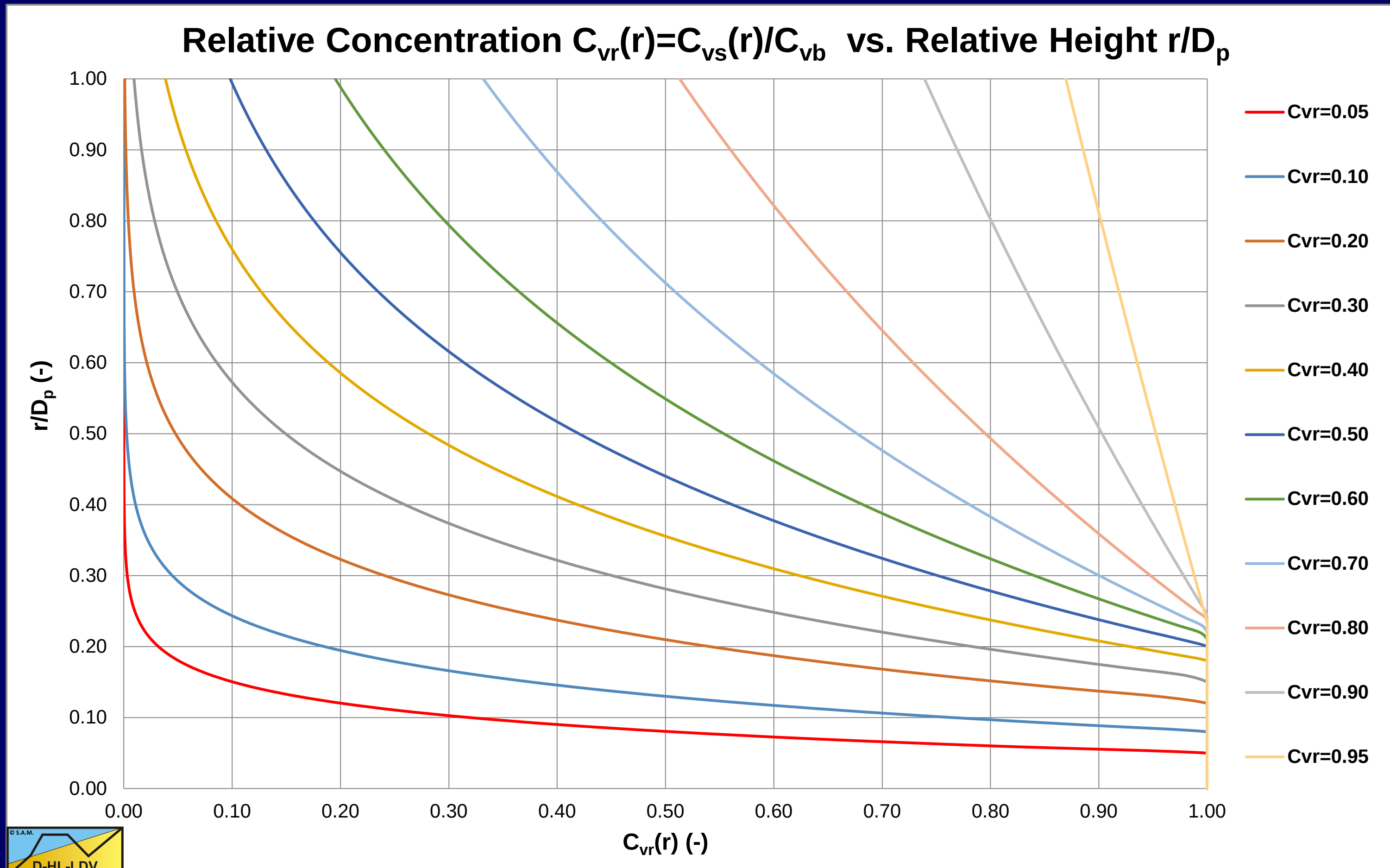
$$C_{vB} = C_{vs} \cdot \frac{\left(\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{tv,ldv}}\right)}{\left(1 - e^{-\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{tv,ldv}}}\right)} = C_{vb} \cdot \frac{\left(\alpha_{sm} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}}\right)}{\left(1 - e^{-\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}}}\right)}$$

$$= C_{vb} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}}$$

$$\alpha_{sm} = \left(1 - e^{-\frac{\alpha_{sm}}{C_{vr}}}\right)$$

$$\alpha_{sm} = 0.9847 + 0.304 \cdot C_{vr} - 1.196 \cdot C_{vr}^2 - 0.5564 \cdot C_{vr}^3 + 0.47 \cdot C_{vr}^4$$

Concentration Distribution at 0.5·LDV



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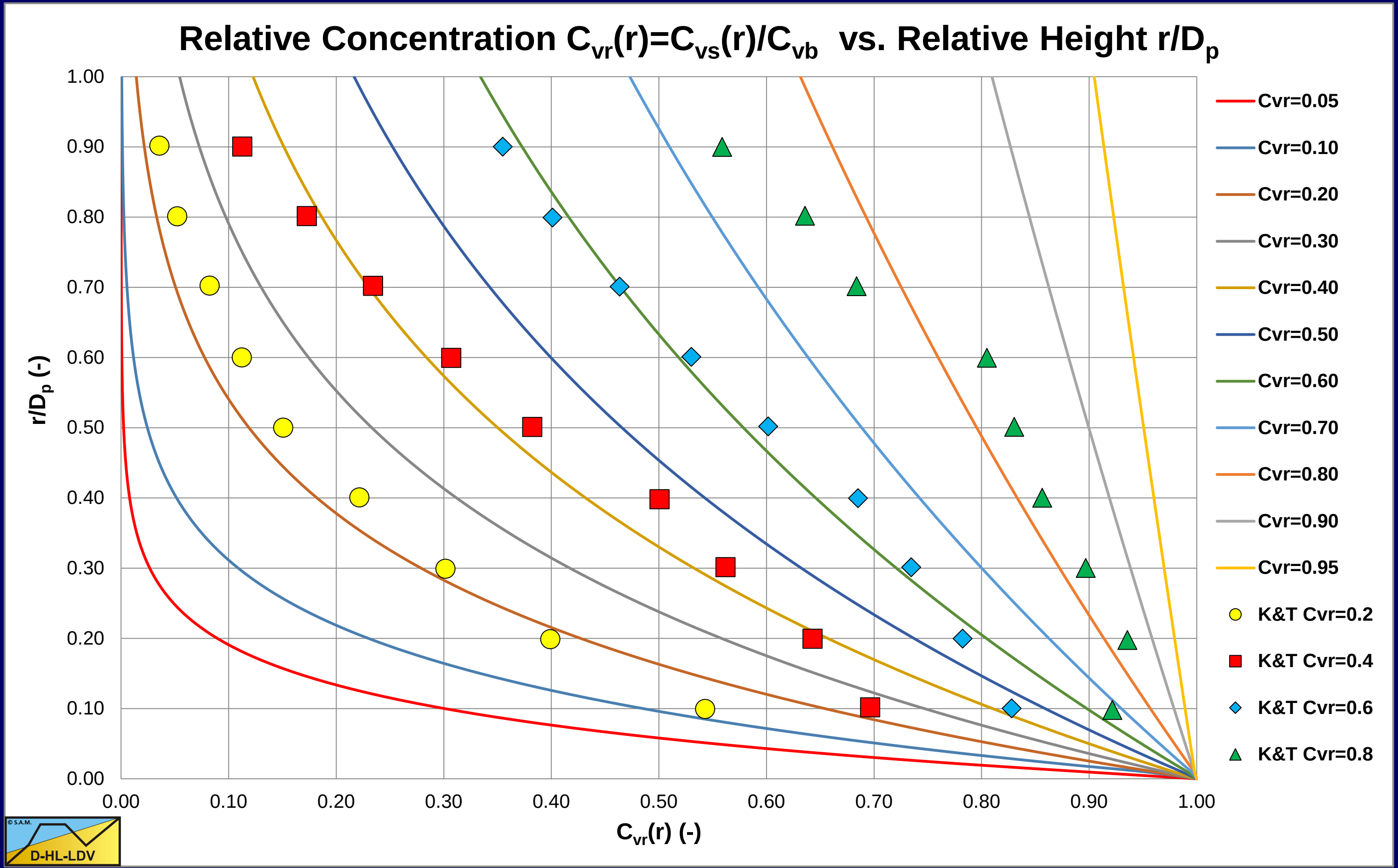
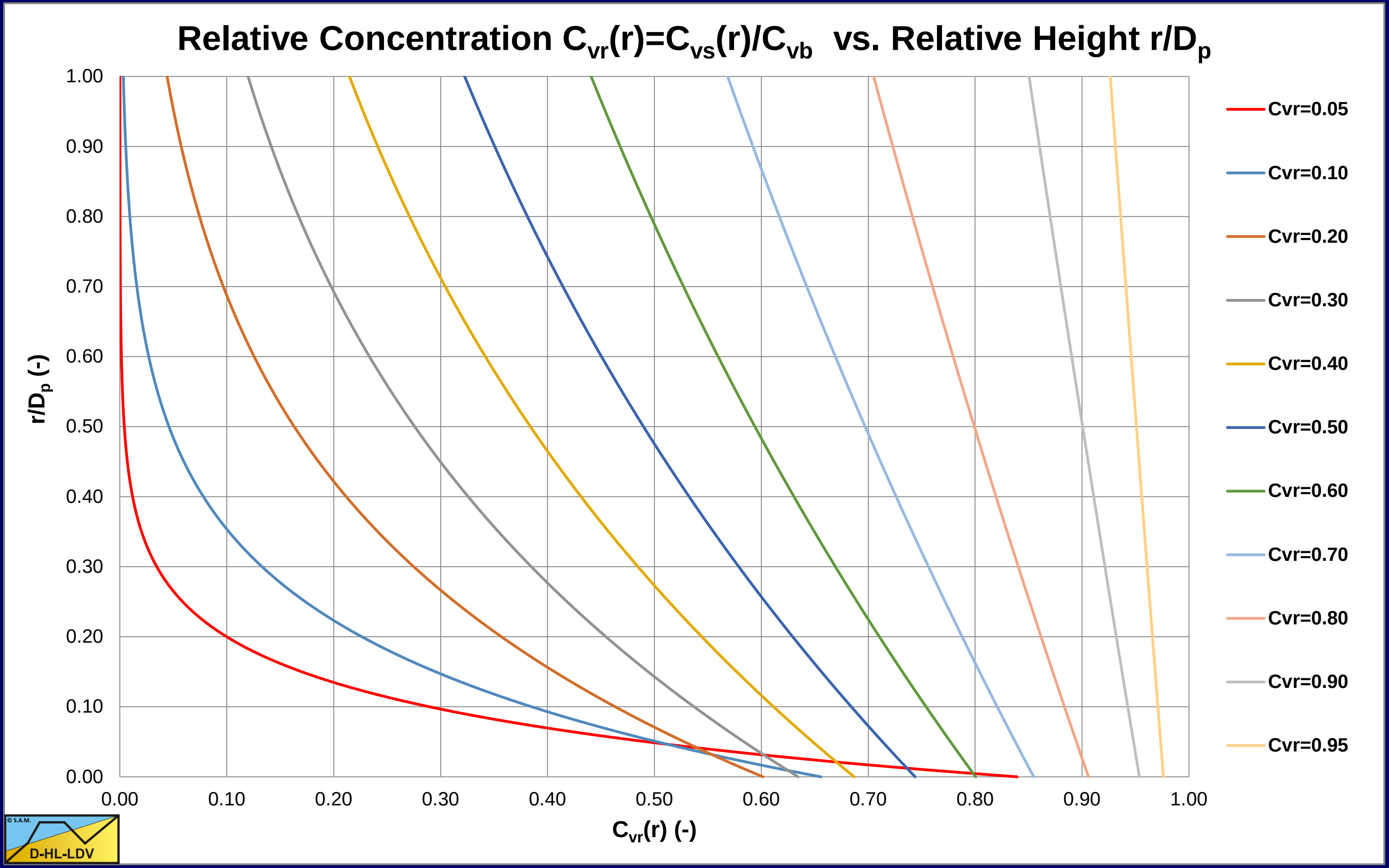
Concentration Distribution at LDV



Kaushal et al. (2005)

Concentration Distribution at 2·LDV



Advection-Diffusion Equation, Modified

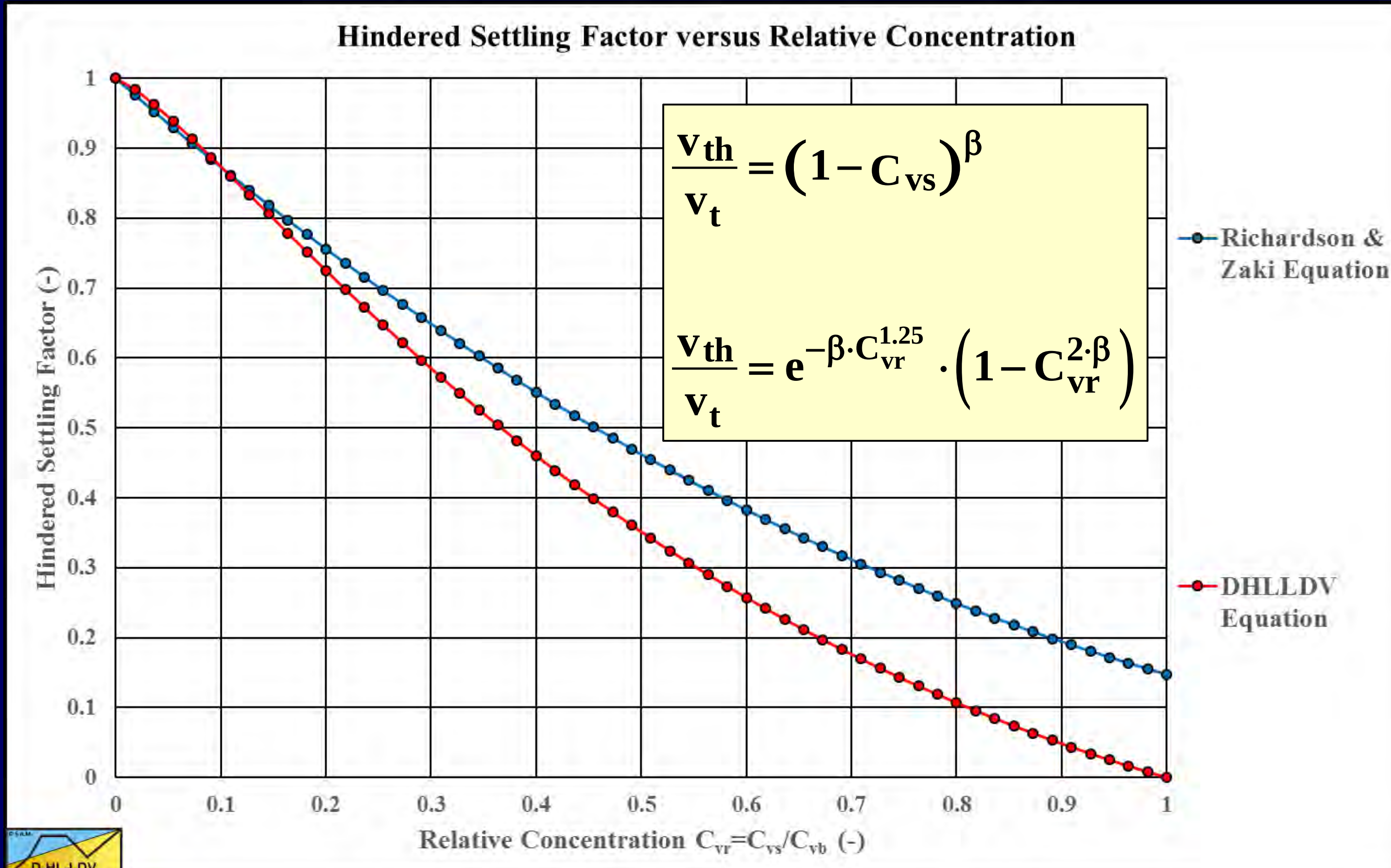
$$C_{vs}(r)\cdot v_{th}(r)+\varepsilon_s\cdot\frac{dC_{vs}(r)}{dr}=C_{vs}(r)\cdot v_{th}(r)+\beta_{sm}\cdot\varepsilon_m\cdot\frac{dC_{vs}(r)}{dr}=0$$

$$\frac{dC_{vs}(r)}{C_{vs}(r)}=-\frac{v_{th}(r)}{\beta_{sm}\cdot\varepsilon_m}\cdot dr \quad\Rightarrow\quad \ln\left(C_{vs}(r)\right)=-\frac{v_{th}(r)}{\beta_{sm}\cdot\varepsilon_m}\cdot r+C$$

$$C_{vs}(r)=C_{vB}\cdot e^{-\frac{v_{th}}{\beta_{sm}\cdot\varepsilon_m}\cdot r}=C_{vB}\cdot e^{-12\cdot\frac{v_{th}}{\beta_{sm}\cdot\kappa\cdot u_*}\cdot\frac{r}{D_p}}$$

- Derived for 2D open channel flow.
- It is assumed the (hindered) terminal settling velocity is constant over the cross section of the pipe.
- The hindered settling equation of Richardson & Zaki gives a velocity above C_{vs}=0.5-0.6 or C_{vr}=1

Hindered Settling



Bed Fraction versus Bed Height

Original Relative Bed Height

$$C_{vs}(r) = C_{vB} \cdot e^{-\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}} \cdot \frac{r}{D_p}}$$

Modified Relative Bed Fraction

$$C_{vs}(r) = C_{vB} \cdot e^{-\frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_*} \cdot \frac{v_{th}}{v_{th,ldv}} \cdot \frac{A_b}{A_p}} \quad \text{with:} \quad f = \frac{A_b}{A_p}$$

Local Hindered Settling, Step 0

Prepare Iteration

$$\frac{dC_{vs,0}(f)}{df} = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot C_{vs,0}(f) \quad \Rightarrow \quad \text{Analytical Solution}$$

$$\frac{dC_{vs,0}(f)}{C_{vb} \cdot df} = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot \frac{C_{vs,0}(f)}{C_{vb}}$$

$$\Rightarrow \quad \frac{dC_{vr,0}(f)}{df} = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot C_{vr,0}(f)$$

$$\Rightarrow \quad \frac{dC_{vr,0}(f)}{df} = -\frac{v_t \cdot e^{-\beta \cdot \left(\frac{C_{vs}}{C_{vb}}\right)^{1.25}} \cdot \left(1 - \left(\frac{C_{vs}}{C_{vb}}\right)^{2 \cdot \beta}\right)}{\beta_{sm} \cdot \varepsilon_m} \cdot C_{vr,0}(f)$$

Local Hindered Settling, Step 1

First Iteration Step

$$\frac{dC_{vs,1}(f)}{df} = -\frac{v_{th,0}(f)}{\beta_{sm} \cdot \varepsilon_m} \cdot C_{vs,0}(f)$$

$$= -\frac{v_t \cdot e^{-\beta \cdot C_{vr,0}(f)^{1.25}} \cdot \left(1 - C_{vr,0}(f)^{2\cdot\beta}\right)}{\beta_{sm} \cdot \varepsilon_m} \cdot C_{vs,0}(f)$$

$$\frac{\left(\dfrac{dC_{vs,1}(f)}{df}\right)}{\left(\dfrac{dC_{vs,0}(f)}{df}\right)} = \frac{\left(v_t \cdot e^{-\beta \cdot C_{vr,0}(f)^{1.25}} \cdot \left(1 - C_{vr,0}(f)^{2\cdot\beta}\right) \cdot C_{vr,0}(f)\right)}{\left(v_t \cdot e^{-\beta \cdot C_{vr}^{1.25}} \cdot \left(1 - C_{vr}^{2\cdot\beta}\right) \cdot C_{vr}\right)}$$

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Local Hindered Settling, Step 2+

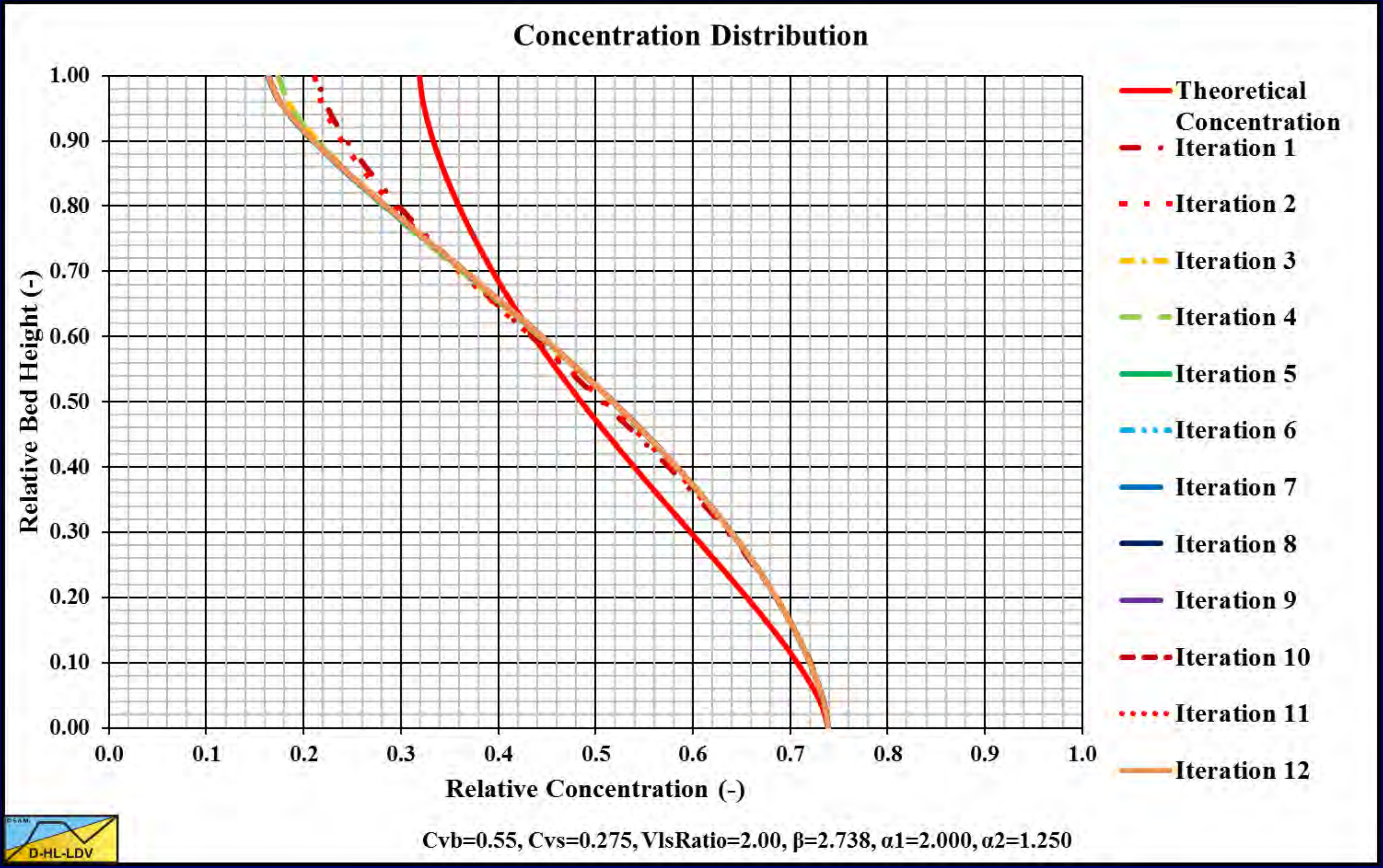
Next Iteration Steps

$$\frac{dC_{vr,i}(f)}{df} = -\frac{v_{th,i-1}(f)}{\beta_{sm}\cdot\varepsilon_m}\cdot C_{vr,i-1}(f)$$

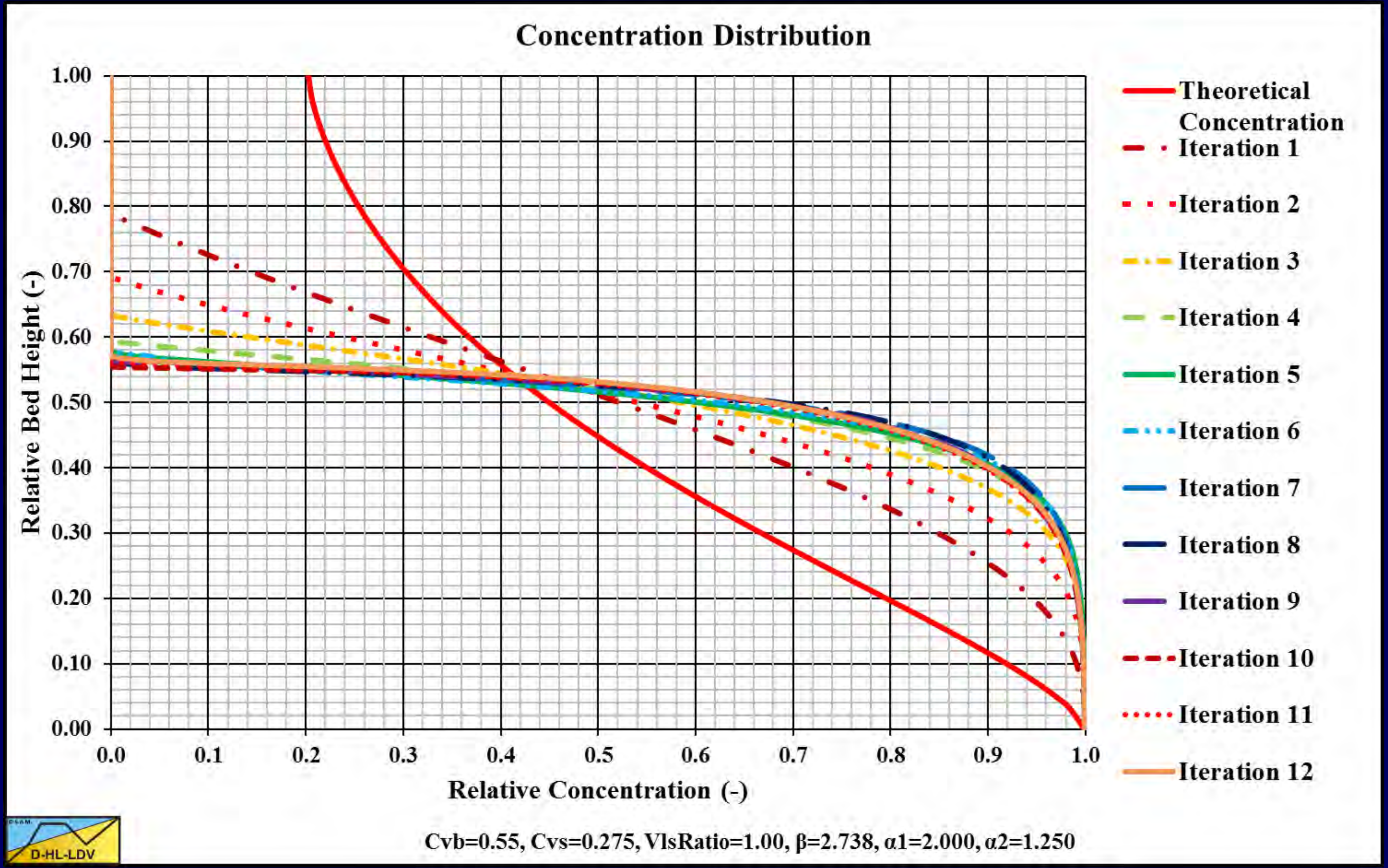
$$= -\frac{v_t\cdot e^{-\beta\cdot C_{vr,i-1}(f)^{1.25}}\cdot\left(1-C_{vr,i-1}(f)^{2\cdot\beta}\right)}{\beta_{sm}\cdot\varepsilon_m}\cdot C_{vr,i-1}(f)$$

$$\frac{\left(\dfrac{dC_{vr,i}(f)}{df}\right)}{\left(\dfrac{dC_{vr,i-1}(f)}{df}\right)} = \frac{\left(v_t\cdot e^{-\beta\cdot C_{vr,i-1}(f)^{1.25}}\cdot\left(1-C_{vr,i-1}(f)^{2\cdot\beta}\right)\cdot C_{vr,i-1}(f)\right)}{\left(v_t\cdot e^{-\beta\cdot C_{vr,i-2}(f)^{1.25}}\cdot\left(1-C_{vr,i-2}(f)^{2\cdot\beta}\right)\cdot C_{vr,i-2}(f)\right)}$$

Iterations High Line Speed , C_{vr}=0.5



Iterations Low Line Speed , $C_{vr}=0.5$



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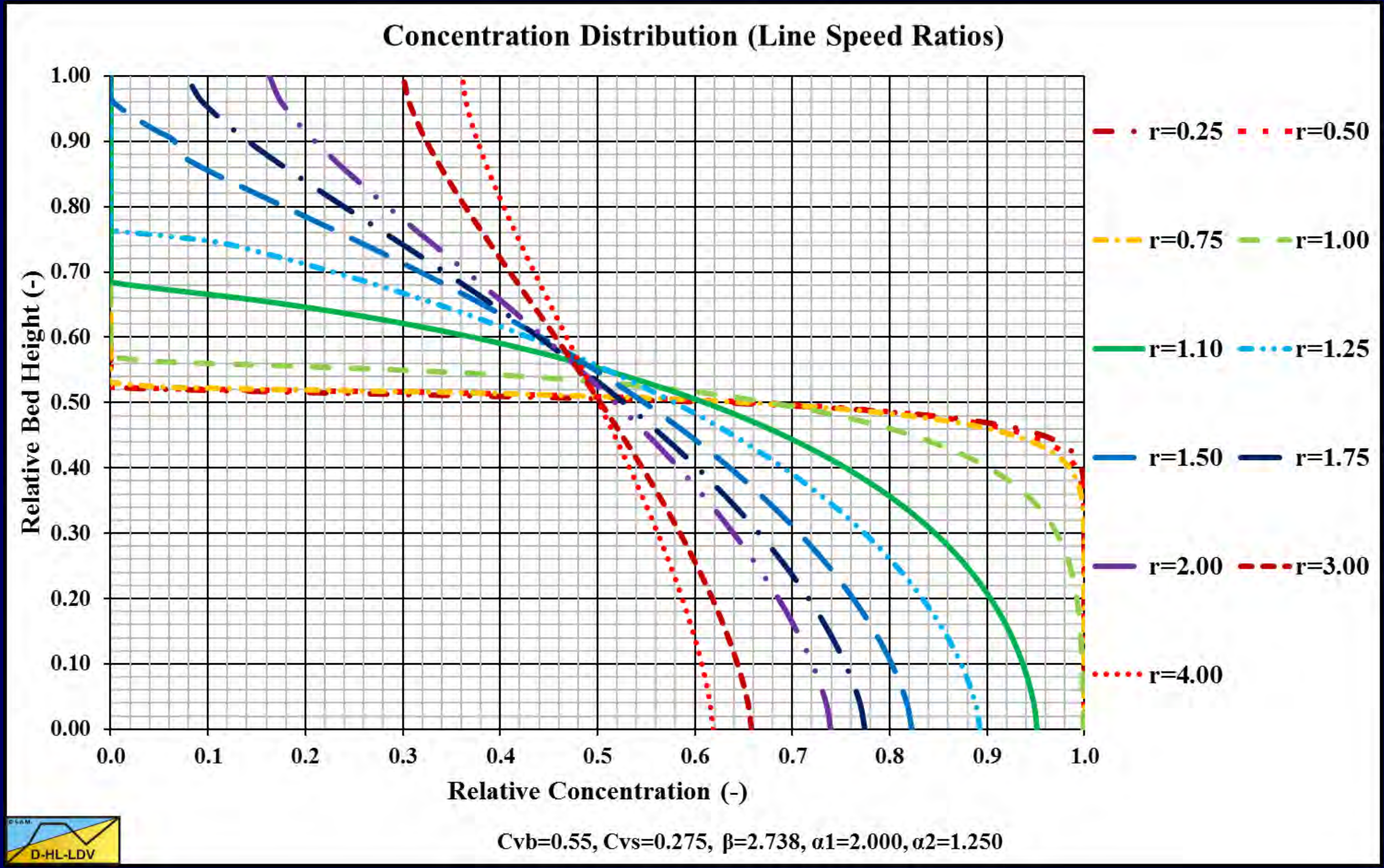
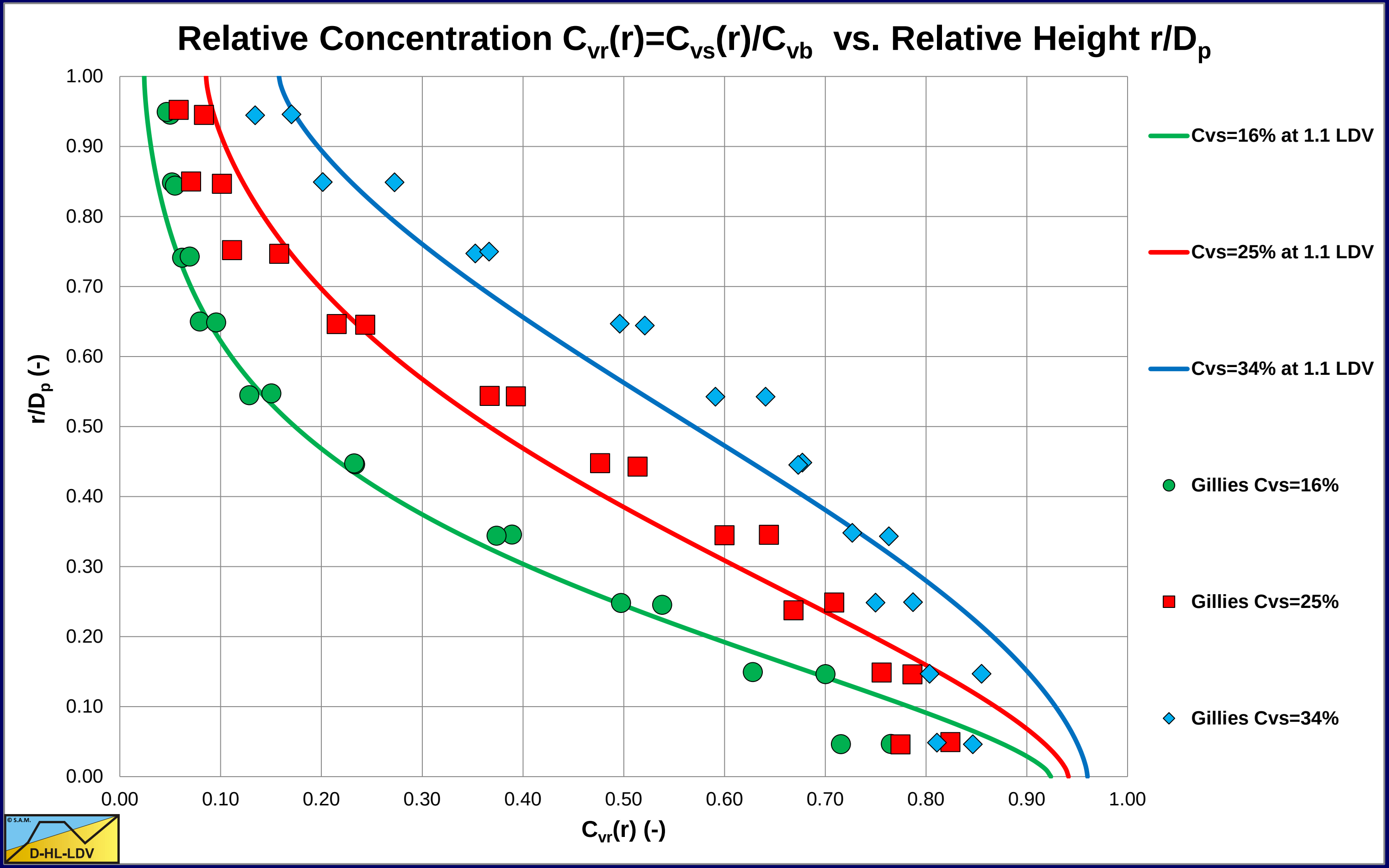
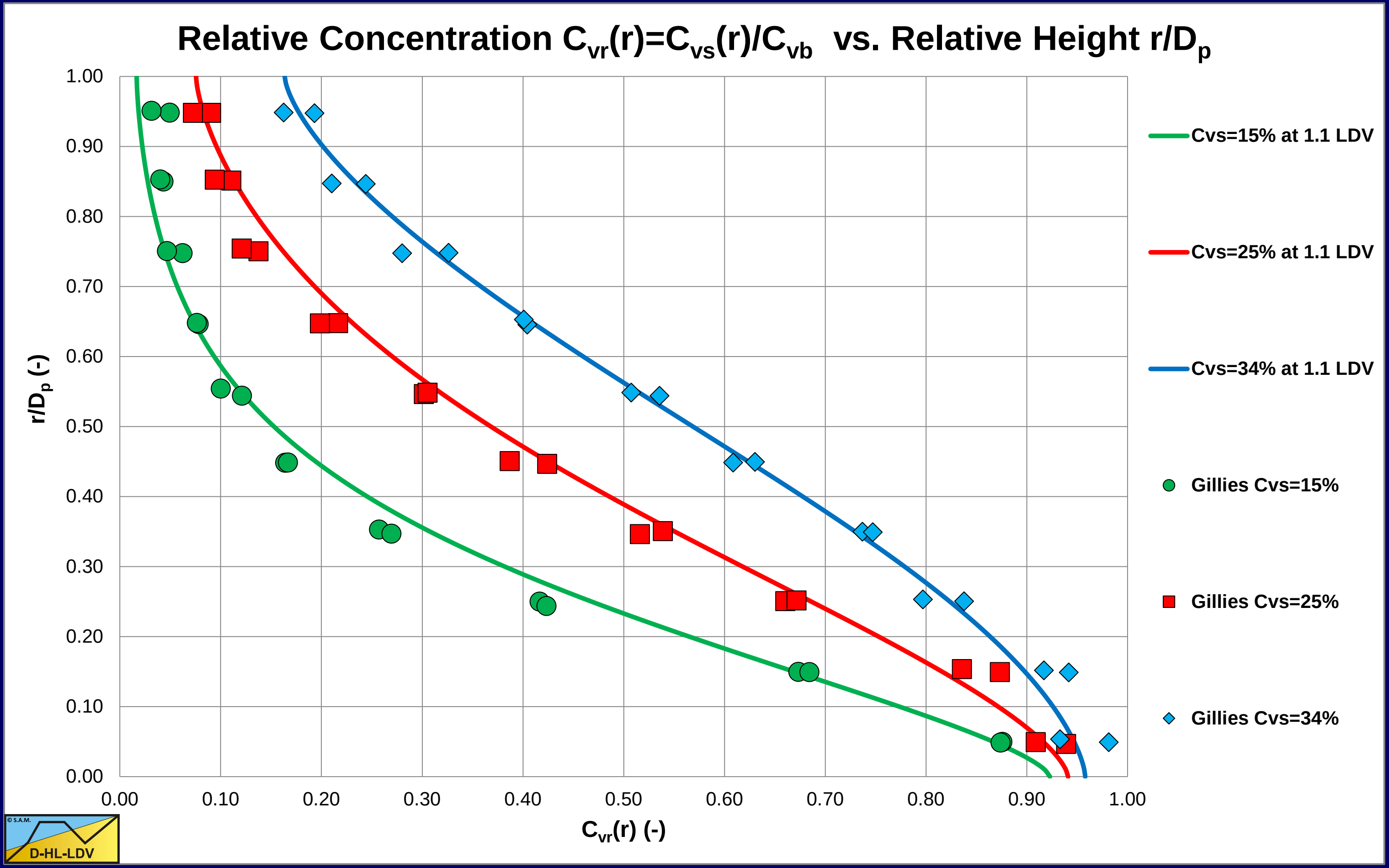
Different Line Speeds, C_{vr}=0.5



Experiments Gillies (1993), d=0.29 mm



Experiments Gillies (1993), d=0.38 mm



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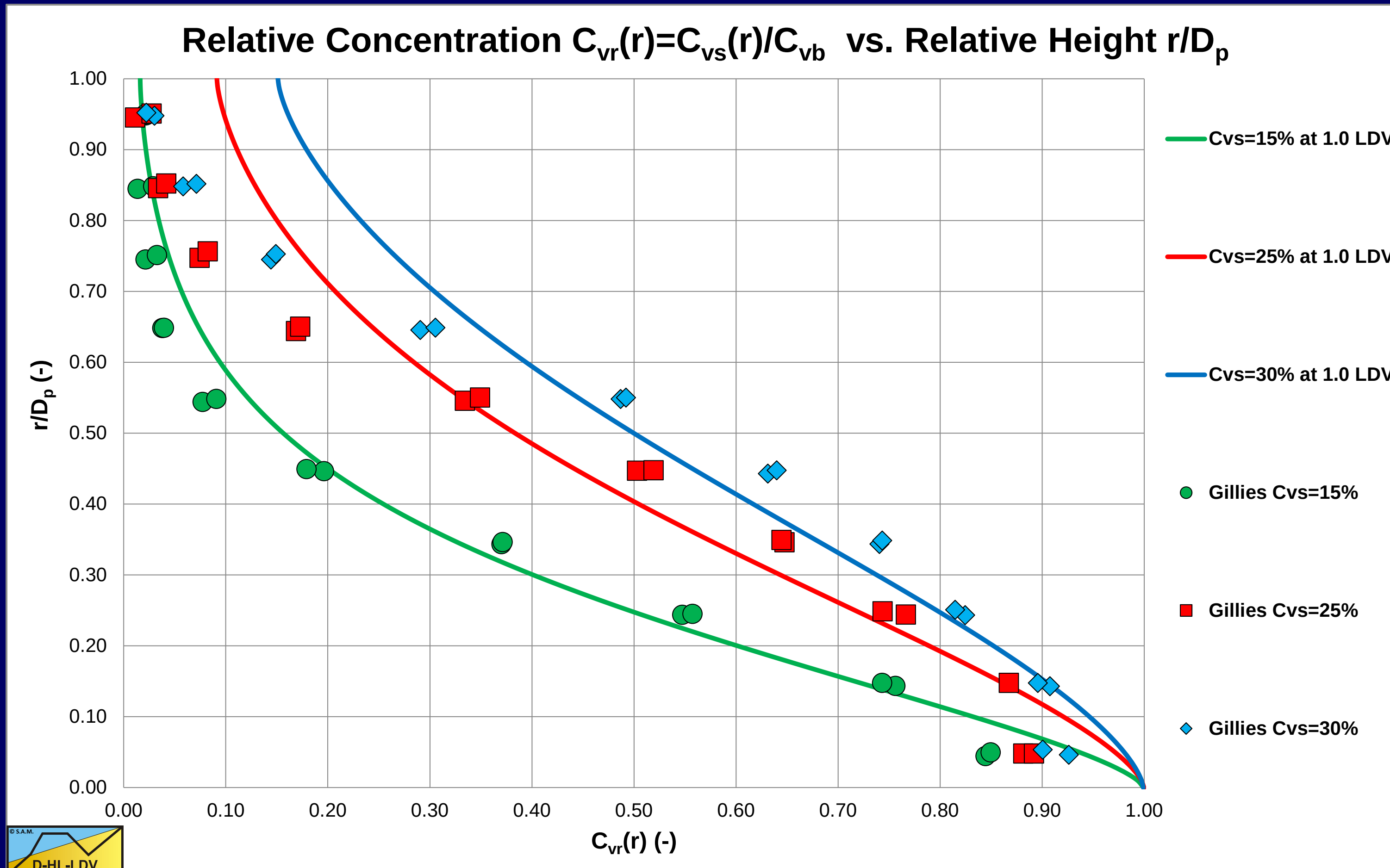
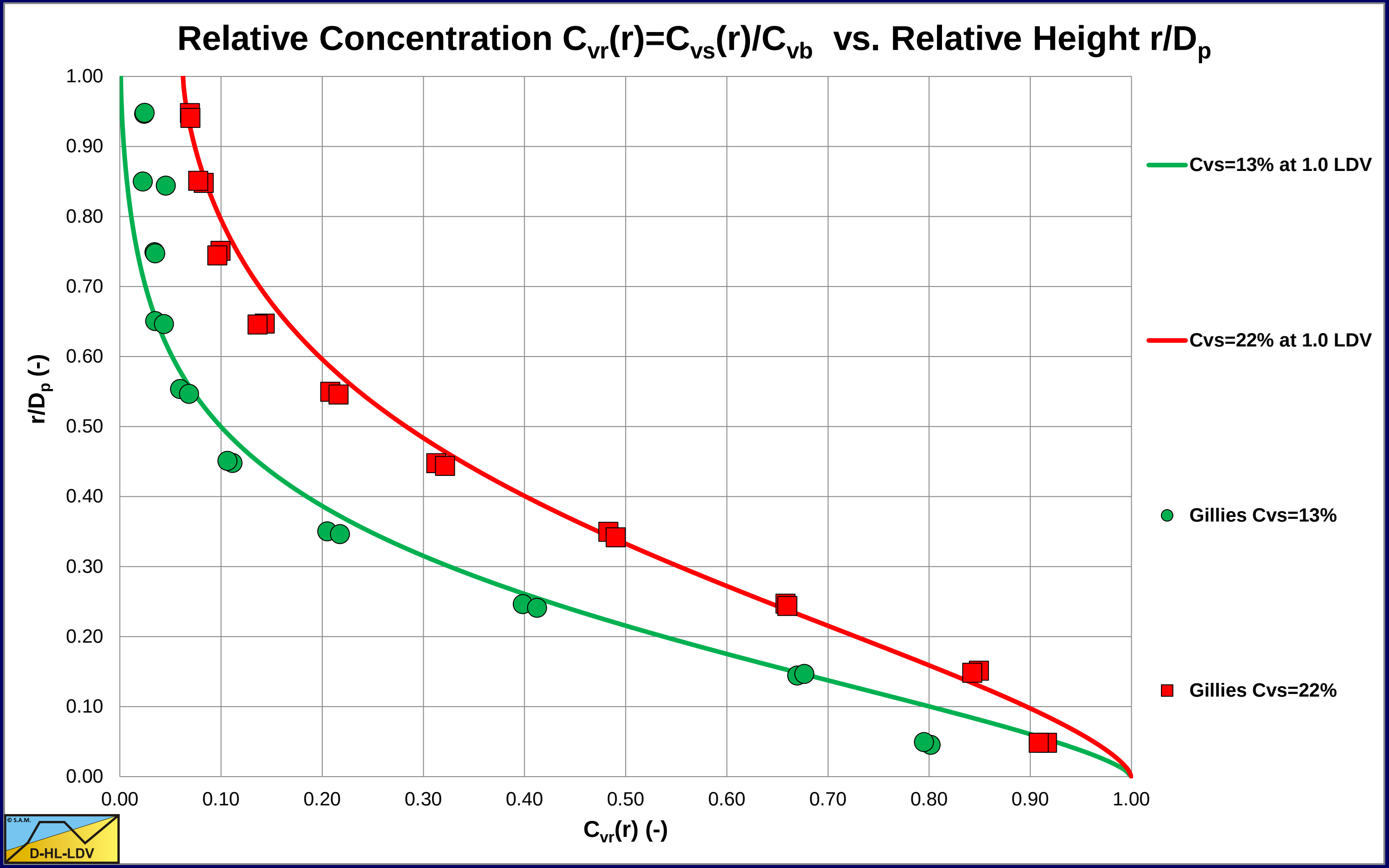
Experiments Gillies (1993), d=0.55 mm



Experiments Gillies (1993), d=2.40 mm



Conclusions

- The concentration distribution using a diffusivity based on the Limit Deposit Velocity gives good results.
- However, the hindered settling equation has to be modified in order to have zero settling velocity at the bed concentration.
- Local hindered settling has to be applied.
- The vertical coordinate has to be replaced by the bed fraction in order to find the correct cross sectional averaged volumetric concentration.

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Questions?