An Analysis of The Hydrostatic Approach of Wilson for the Friction of a Sliding Bed

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GLDD SAFETY WEEK 2014

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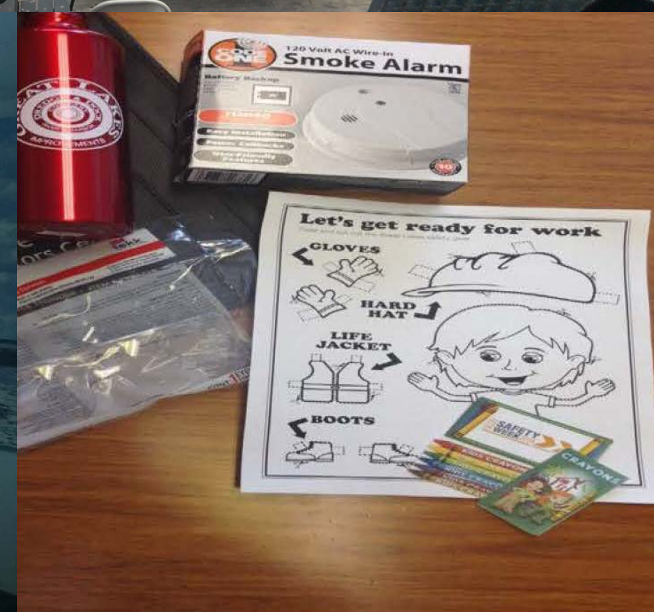
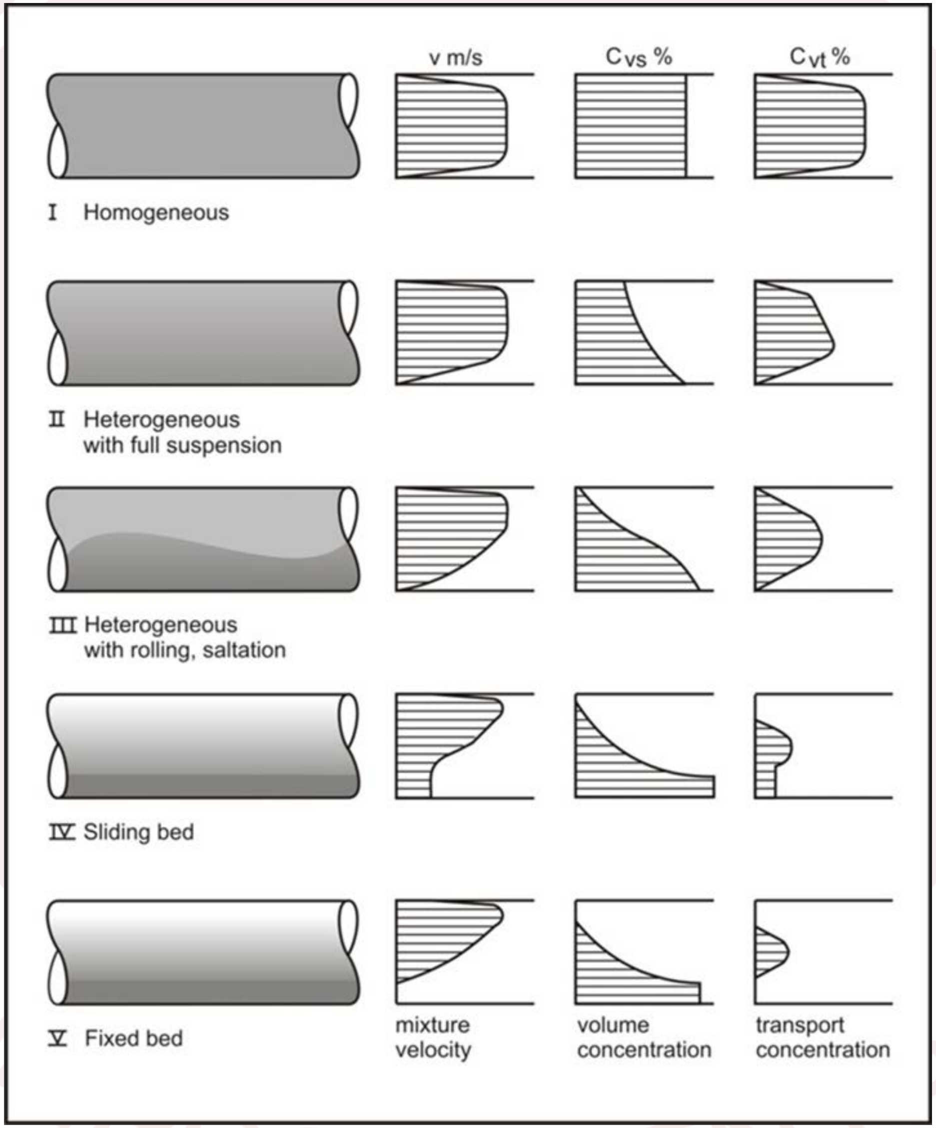


Problem Definition

- In fully stratified particle transport, the resistance to flow consists of the friction of the fluid above the bed plus the friction resistance of the bed on the pipewall.
- In the commonly used Wilson model, the bed friction is calculated assuming a hydrostatic normal stress distribution between the bed and the pipe wall.
- Problem: For high concentrations, is the very large resistance implied by the Wilson approach correct?
- Here we analyze two alternative approaches and suggest which we feel is correct.

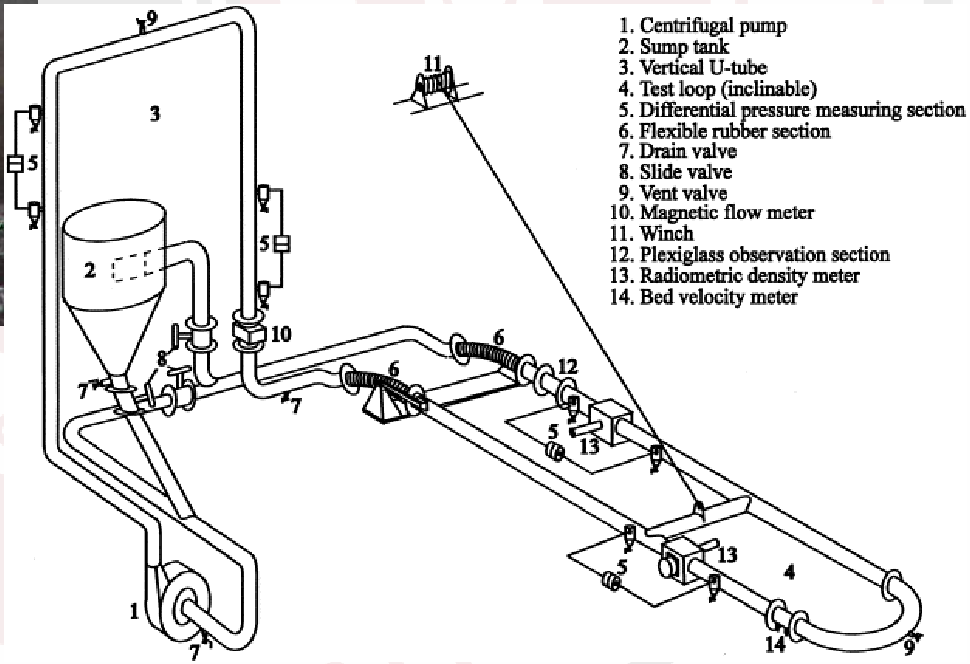
Regimes Overview



Volume Concentrations

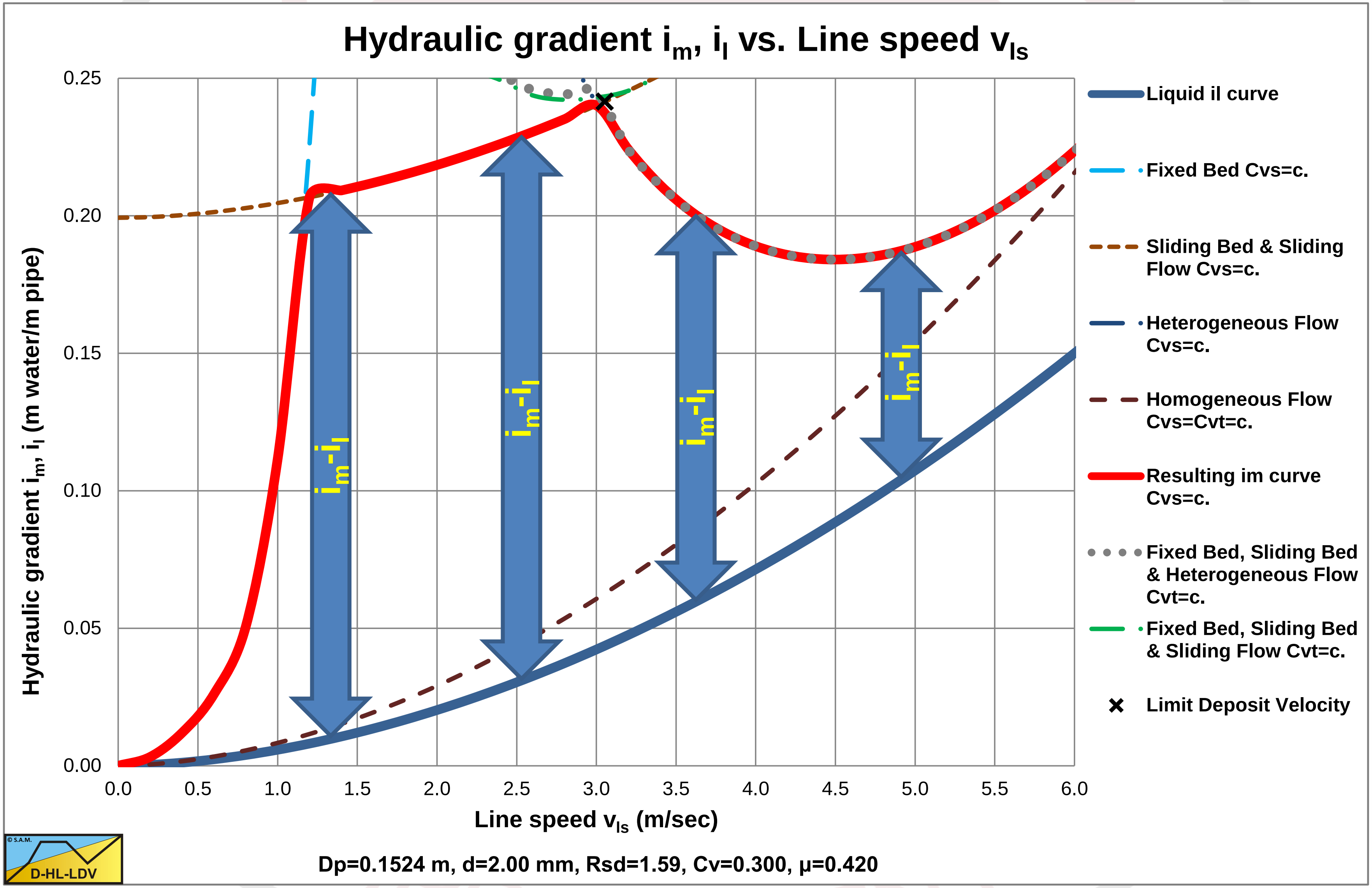
$C_v = \frac{\rho_m - \rho_{fl}}{\rho_s - \rho_{fl}}$: Volume concentration from density readings

$C_{vt} = \frac{\dot{v}_s}{\dot{v}_m}$: Transport (delivered) volume concentration.

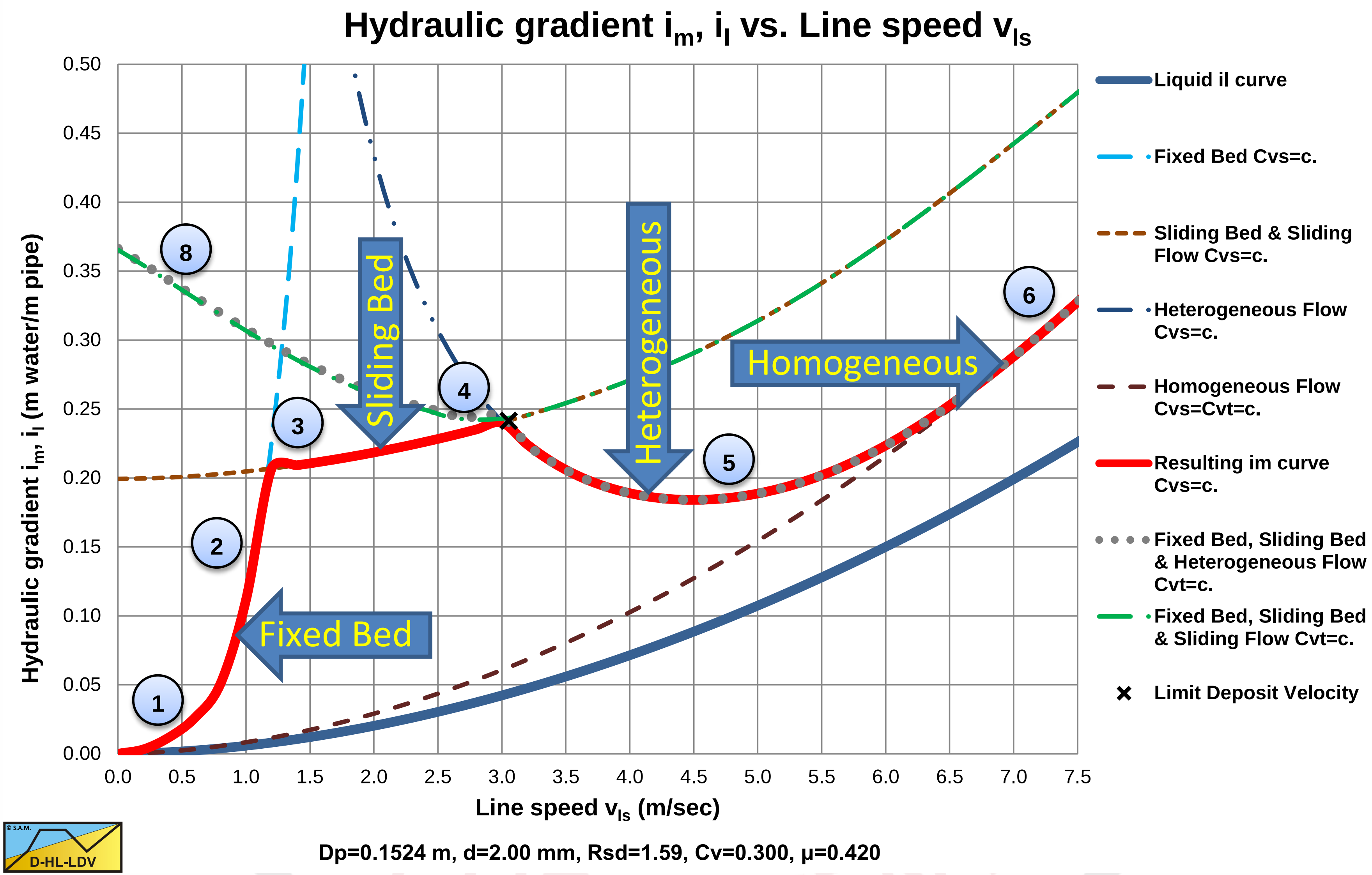


$C_{vs} = \frac{Q_s}{Q_m}$: Spatial volume concentration. Generally fixed in a lab environment.

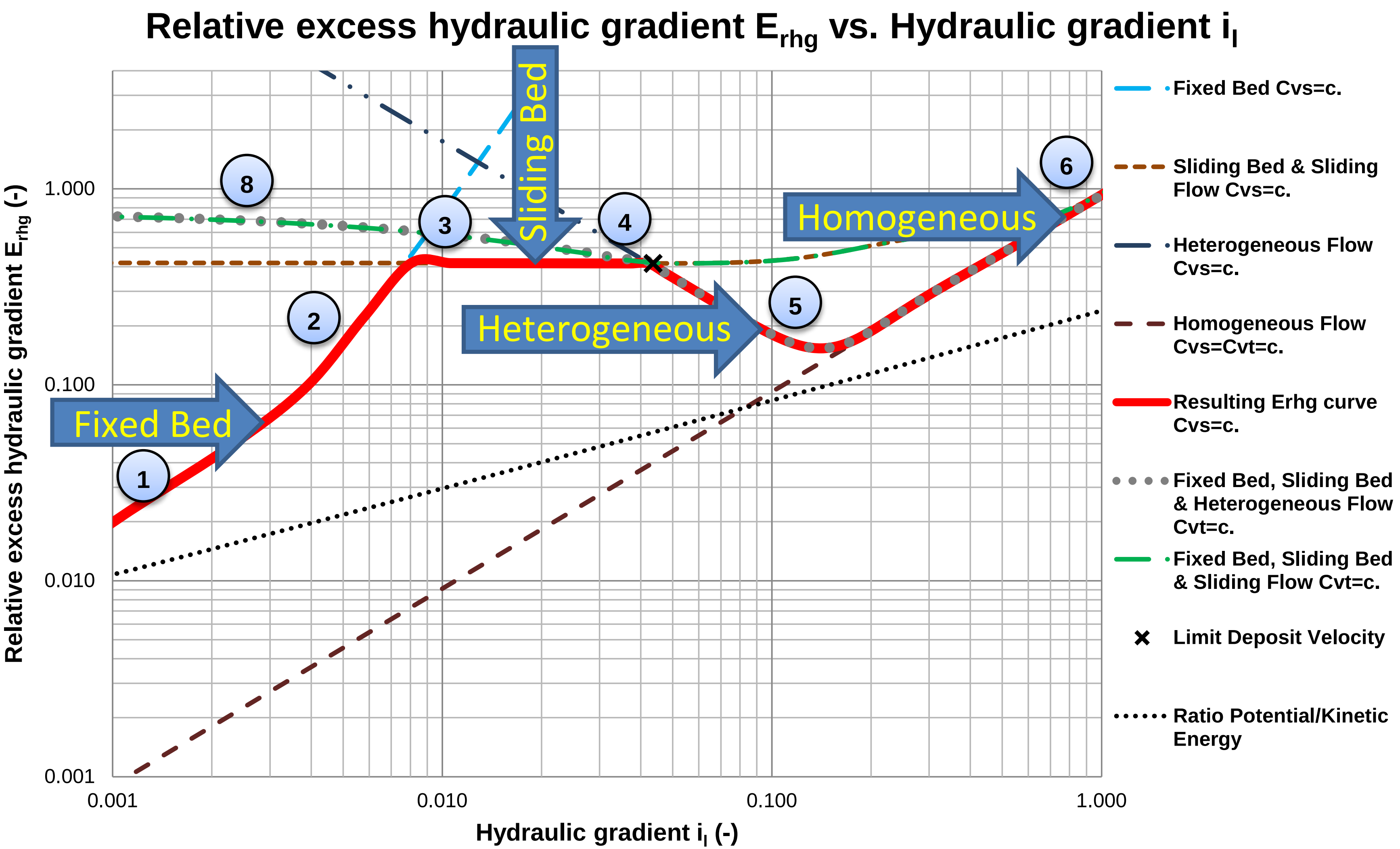
The Excess Hydraulic Gradient



Regimes Overview i_m-v_{ls}

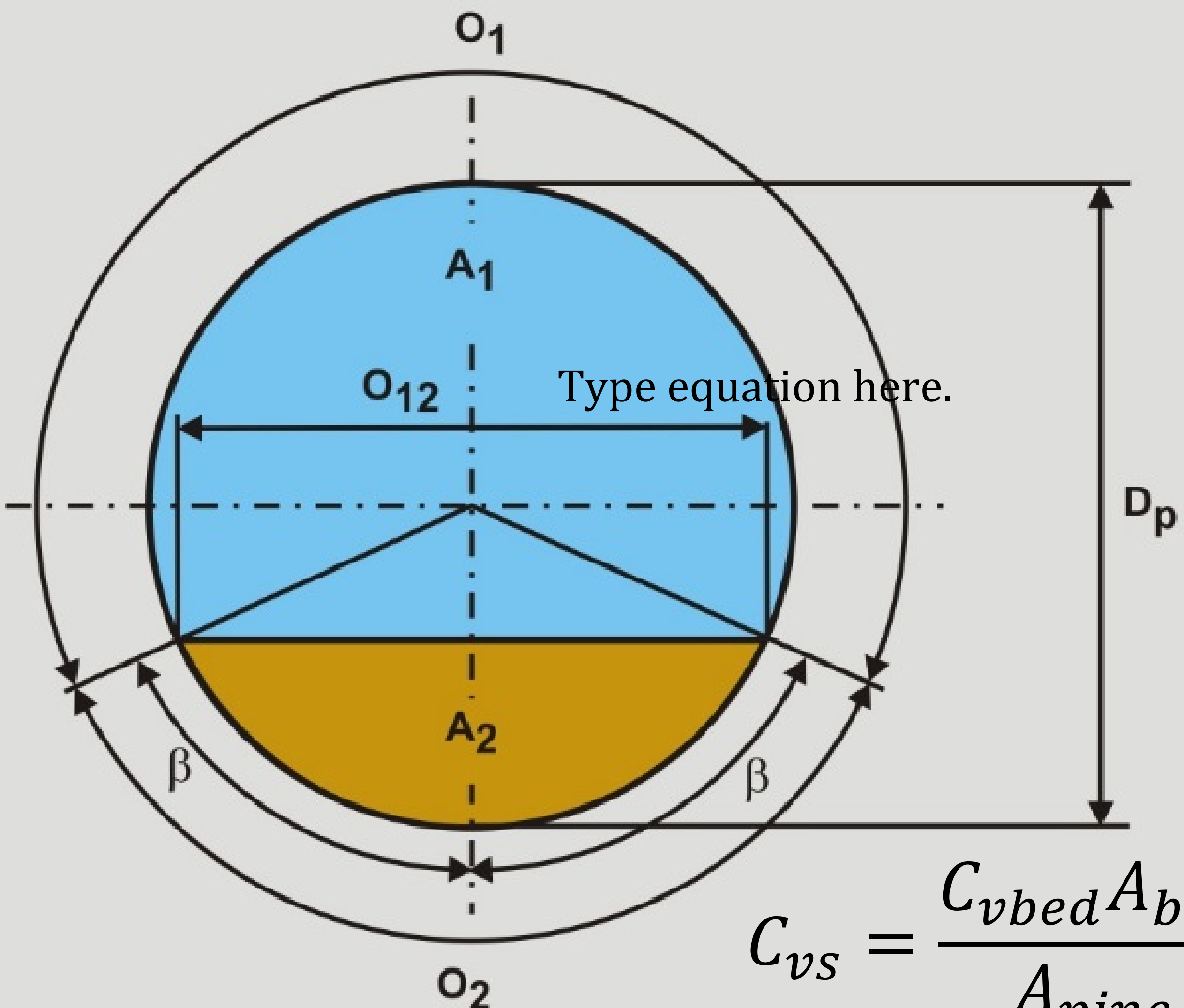


Regimes Overview E_{rhg}-i_l



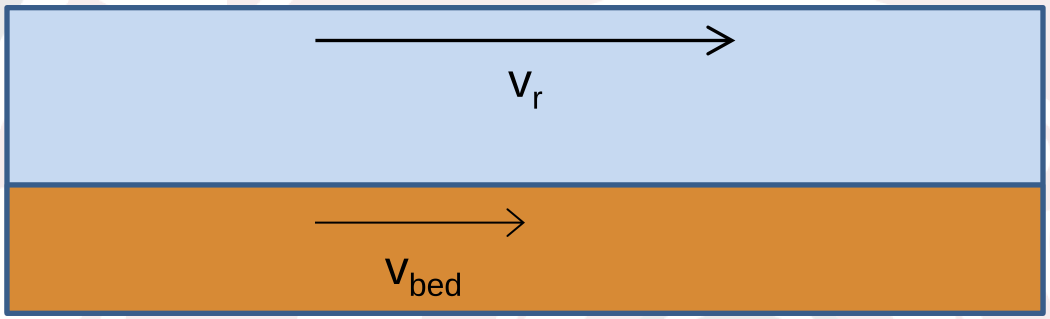
Dp=0.1524 m, d=2.00 mm, Rsd=1.59, Cv=0.300, μ=0.420

The Geometry



$$C_{vs} = \frac{C_{vbed} A_{bed}}{A_{pipe}}$$

The Transport of Fluid and Material



$v_r \cdot A_r$

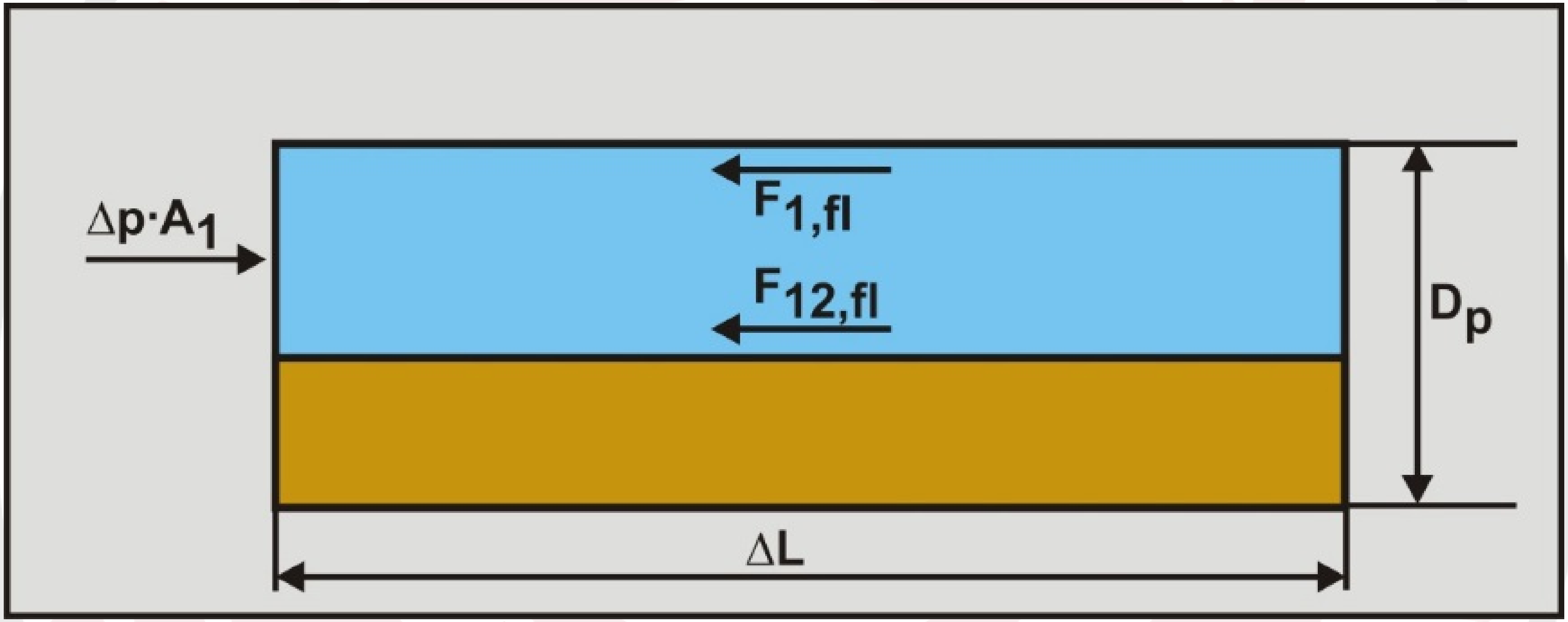
$v_{bed} \cdot A_{bed}$

$$Total\ Flow = v_r \cdot A_r + v_{bed} \cdot A_{bed}$$

$$v_{ls} = \frac{v_r \cdot A_r + v_{bed} \cdot A_{bed}}{A_r + A_{bed}} = \frac{v_r \cdot A_r + v_{bed} \cdot A_{bed}}{A_{pipe}}$$

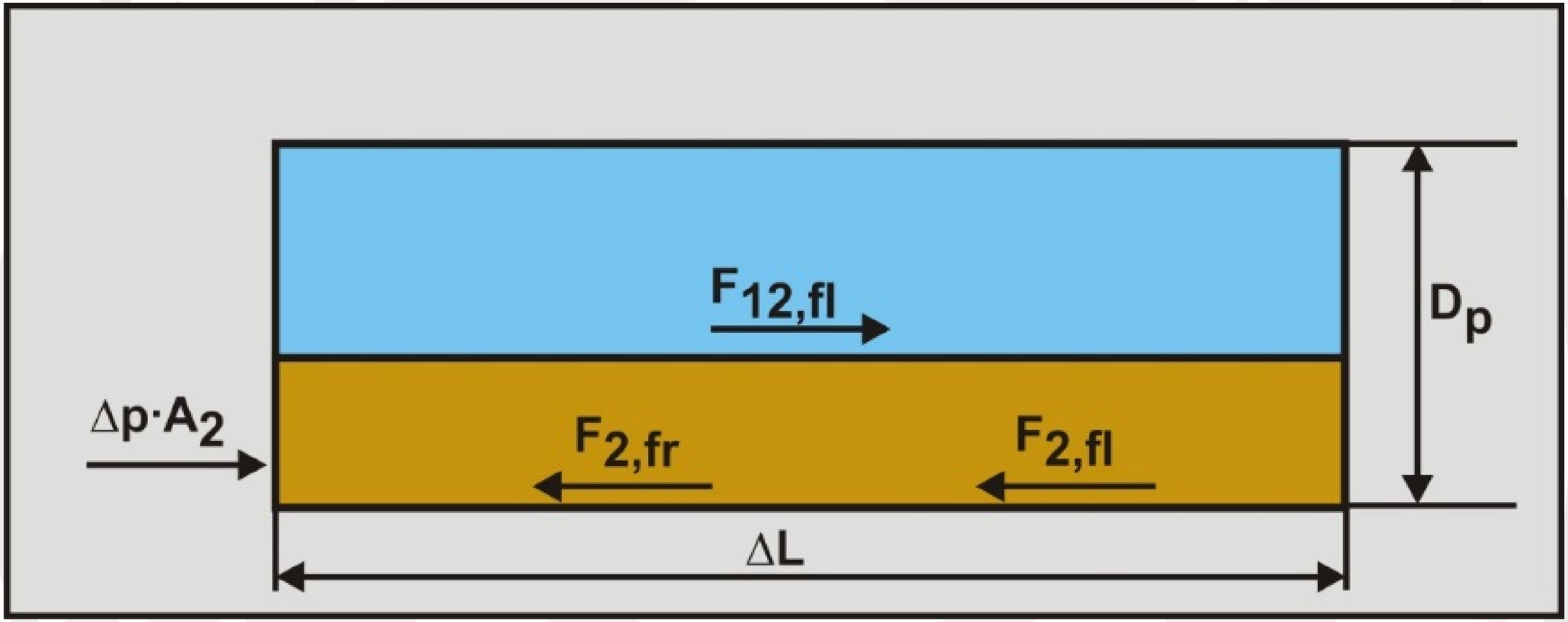
$$C_{vt} = \frac{C_{vbed} V_{bed} A_{bed}}{V_{ls} A_{pipe}}$$

The Forces Above the Bed



$$\Delta p \cdot A_1 = F_{1,fl} + F_{12,fl}$$

The Forces on the Bed



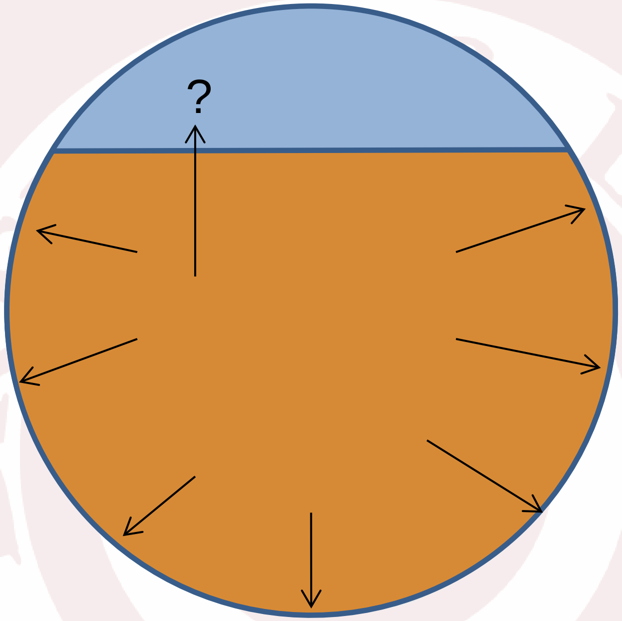
$$\Delta p \cdot A_2 + F_{12,fl} = F_{2,fr} + F_{2,fl}$$

3 Approaches to $F_{2,fr}$

1. The Wilson Hydrostatic Approach
2. The Weight Approach
3. The Normal Stress Carrying The Weight Approach

$$i_m - i_{fl} = \frac{\mu_{fr} \cdot F_?}{A_p \cdot \rho_{fl} \cdot g \cdot \Delta L}$$

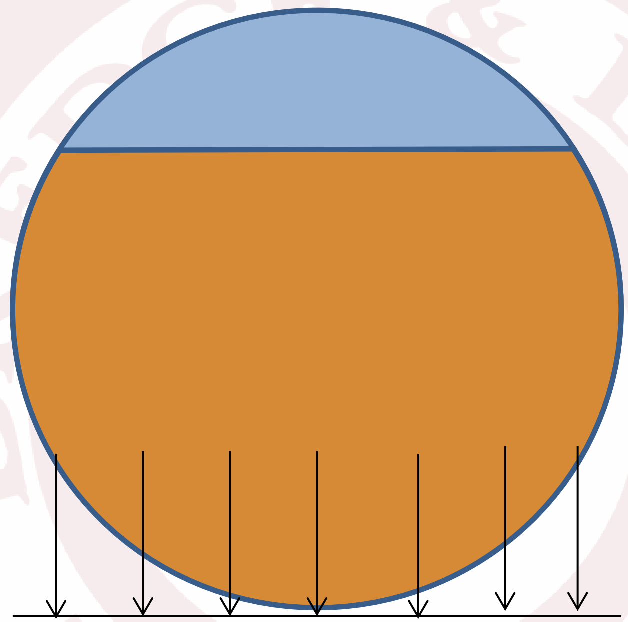
Wilson Approach



Wilson et al assume a hydrostatic normal force distribution on the pipe wall.

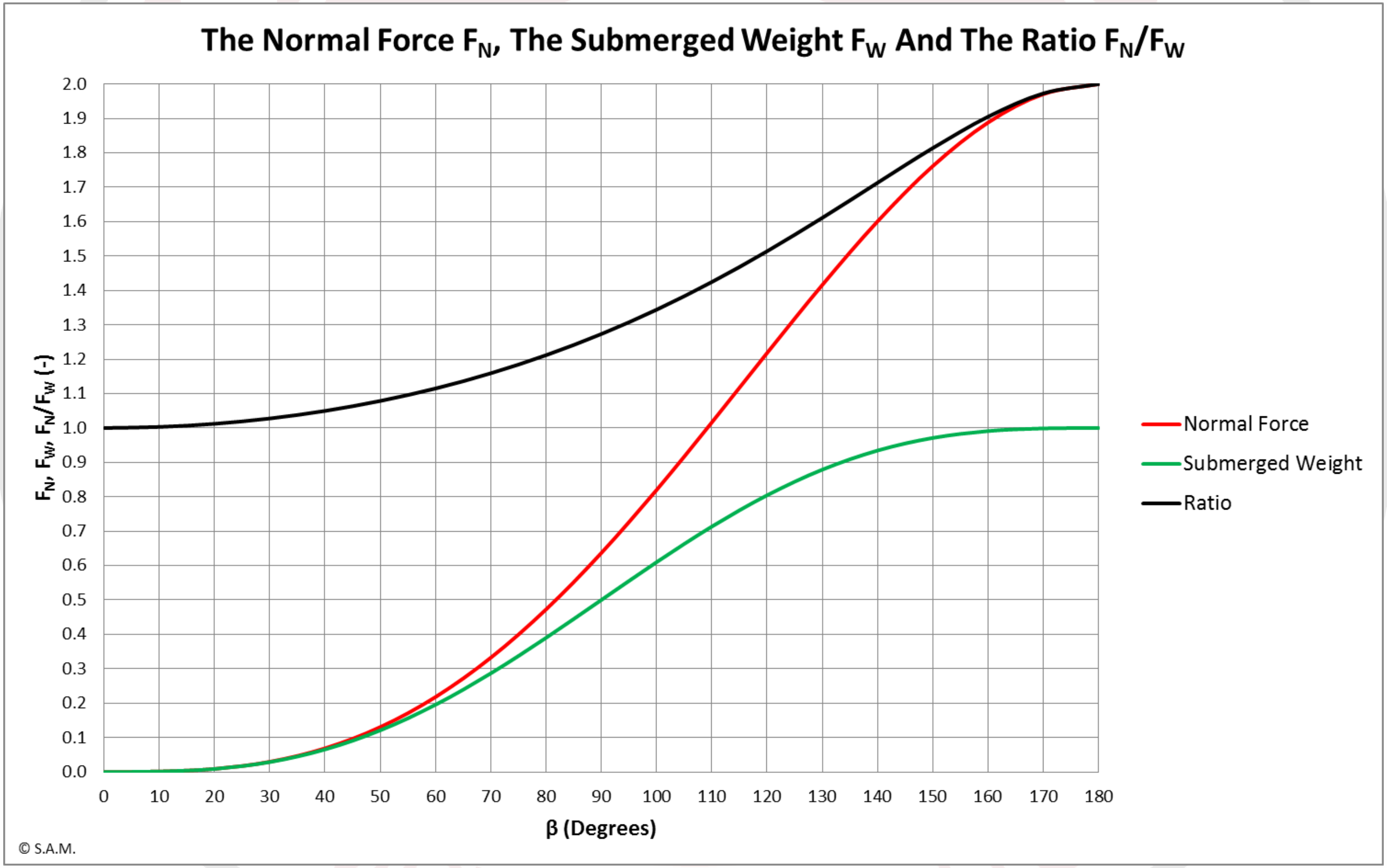
$$F_n = \rho_{fl} \cdot g \cdot L \cdot R_{sd} \cdot C_{vb} \cdot \frac{{D_p}^2}{2} \cdot (\beta - sin\beta \cdot cos\beta)$$

Submerged Weight Approach

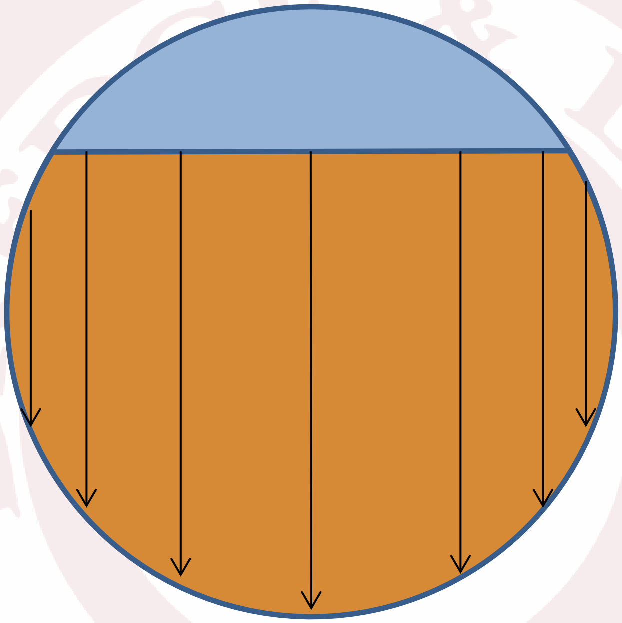


$$F_w = \rho_{fl} \cdot g \cdot L \cdot R_{sd} \cdot C_{vb} \cdot \frac{D_p^2}{4} \cdot \left(\beta - \sin(\beta) \cdot \cos(\beta)\right)$$

Wilson and Submerged Weight Approaches



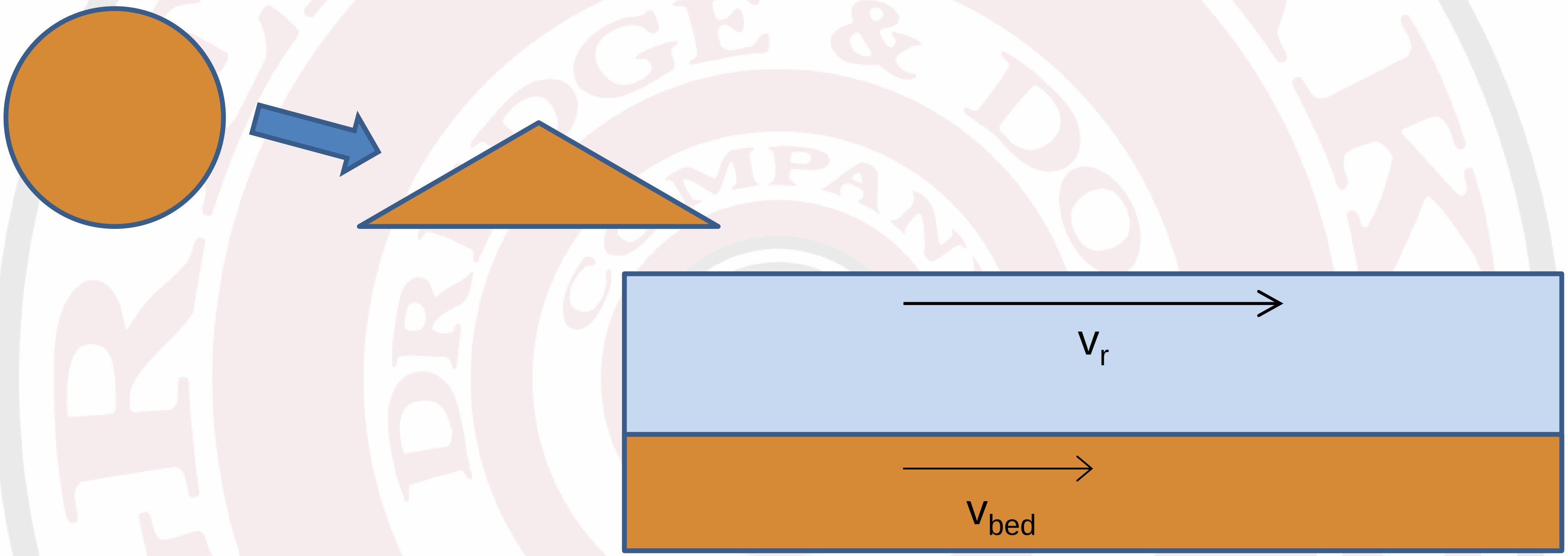
The Normal Force Carrying the Weight



For β>π/2:

$$F_n = 2\cdot\rho_{fl}\cdot R_{sd}\cdot g\cdot C_{vb}\cdot R^2\cdot\left(\sin(\beta)-\beta\cdot\cos(\beta)\right) = 2\cdot\rho_{fl}\cdot R_{sd}\cdot g\cdot C_{vb}\cdot A_p\cdot\frac{\left(\sin(\beta)-\beta\cdot\cos(\beta)\right)}{\pi}$$

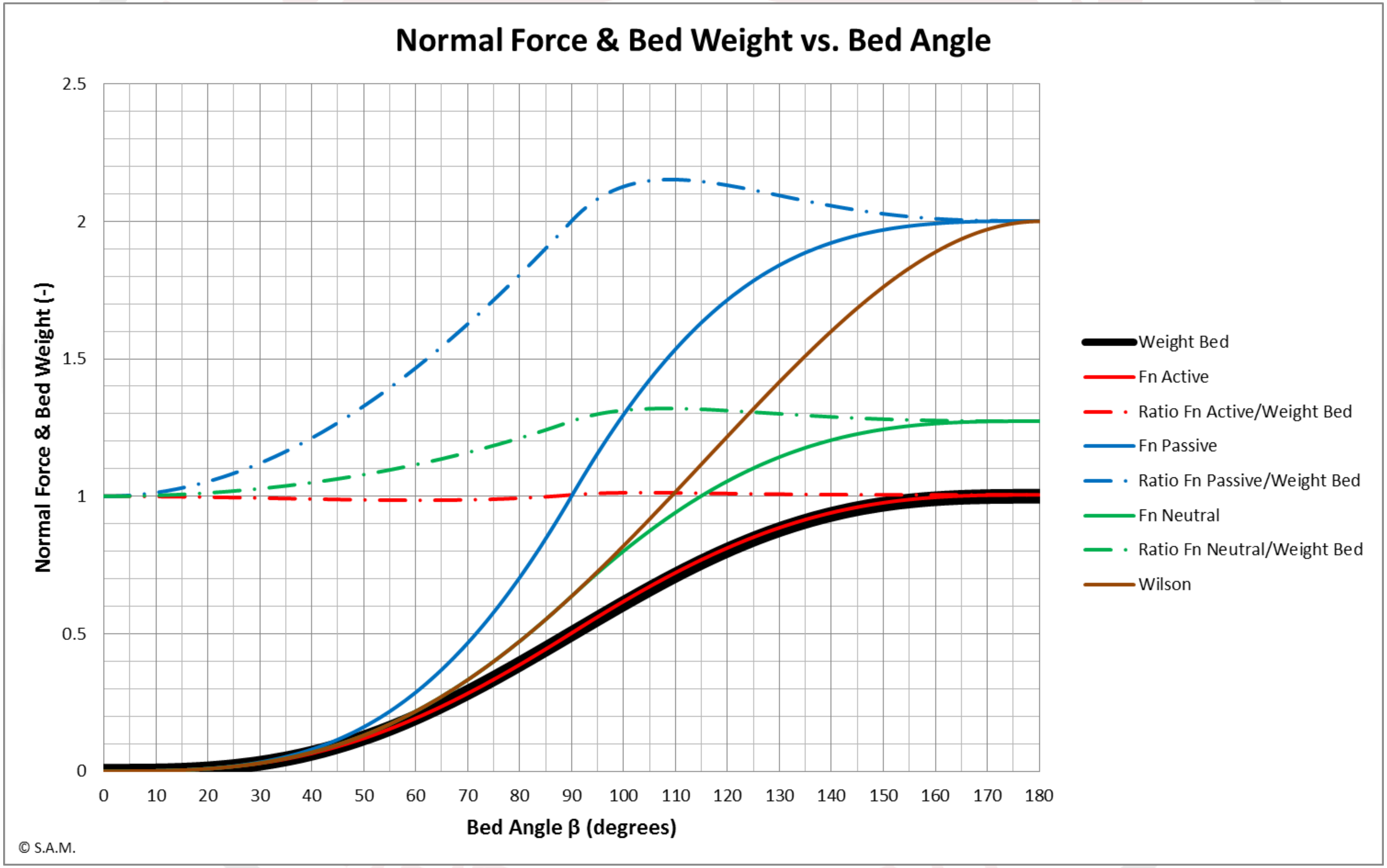
Active/Passive failure approach



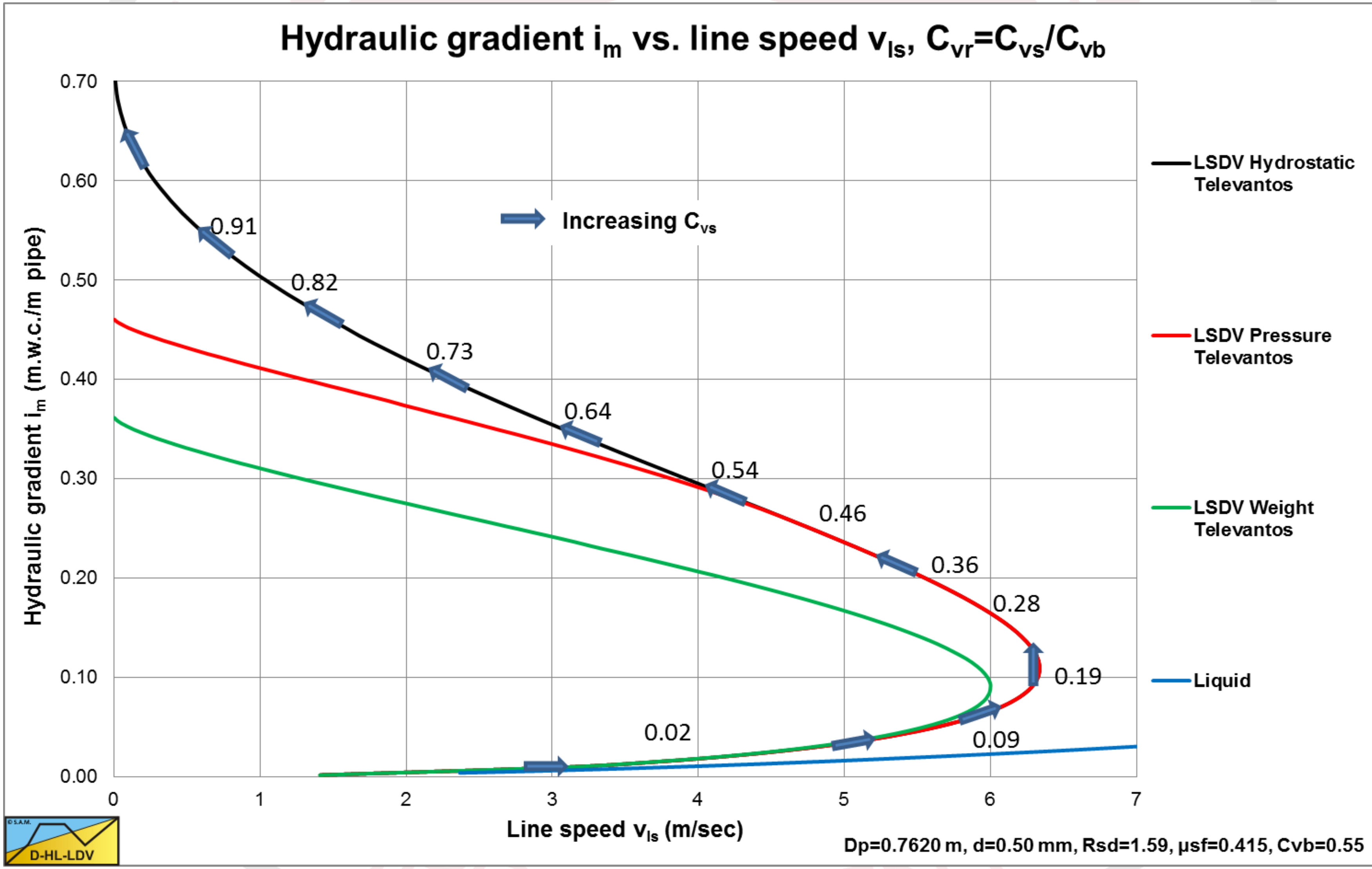
For β>π/2:

$$F_n = 2 \cdot \rho_{fl} \cdot R_{sd} \cdot g \cdot C_{vb} \cdot R^2 \cdot \left(\begin{array}{l} \frac{K-1}{3} \cdot \sin^3(\beta) + \sin(\beta) - \frac{K+1}{2} \cdot (\pi - \beta) \cdot \cos(\beta) - \frac{K-1}{2} \cdot \sin(\beta) \cdot \cos^2(\beta) \\ + \frac{2}{3} \cdot (K-1) + 2 - \frac{2}{3} \cdot (K-1) \cdot \sin^3(\beta) - 2 \cdot \sin(\beta) \end{array} \right)$$

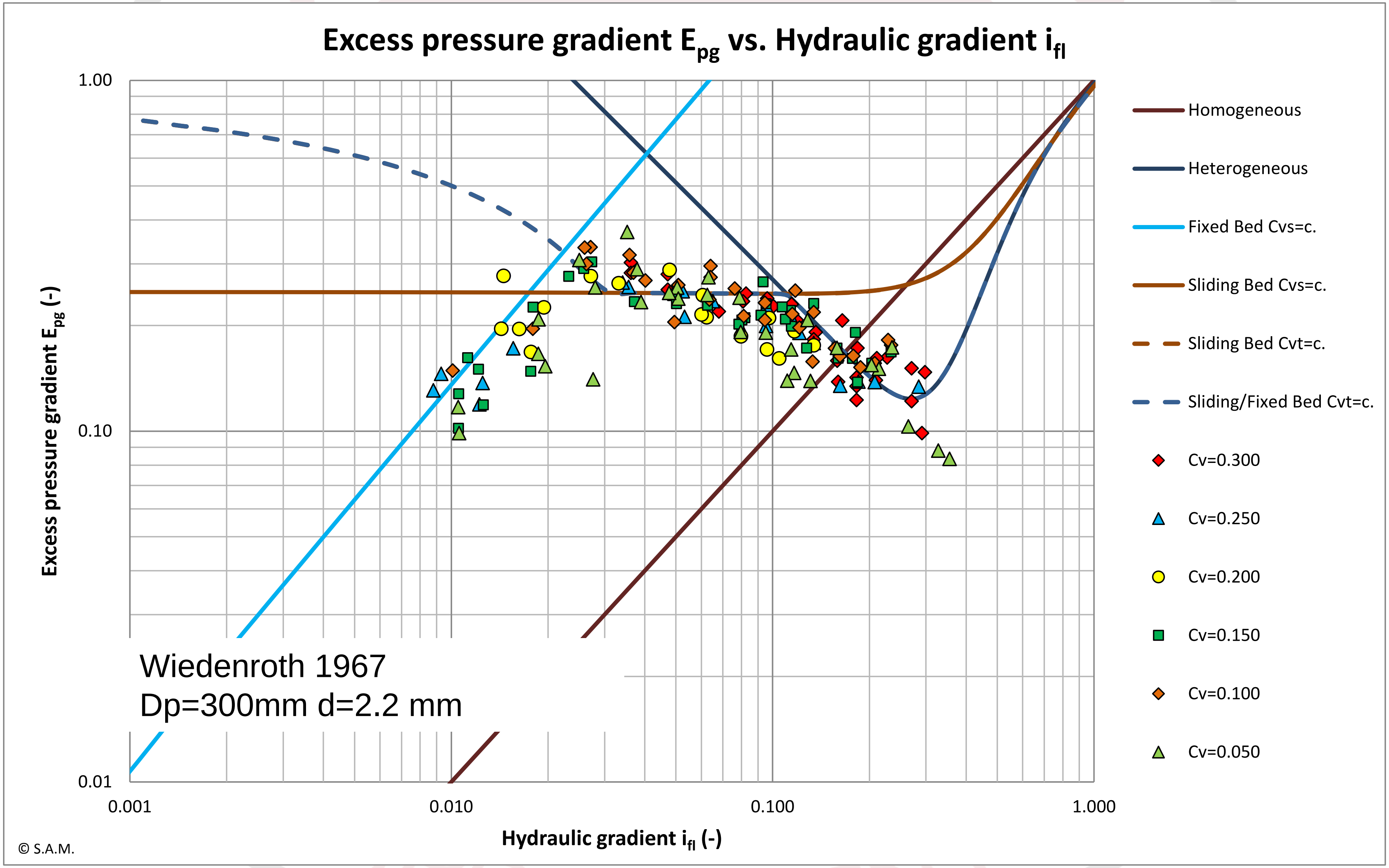
Active/Passive Failure Approach



Comparing the 3 approaches – Fixed/Sliding Bed Transition

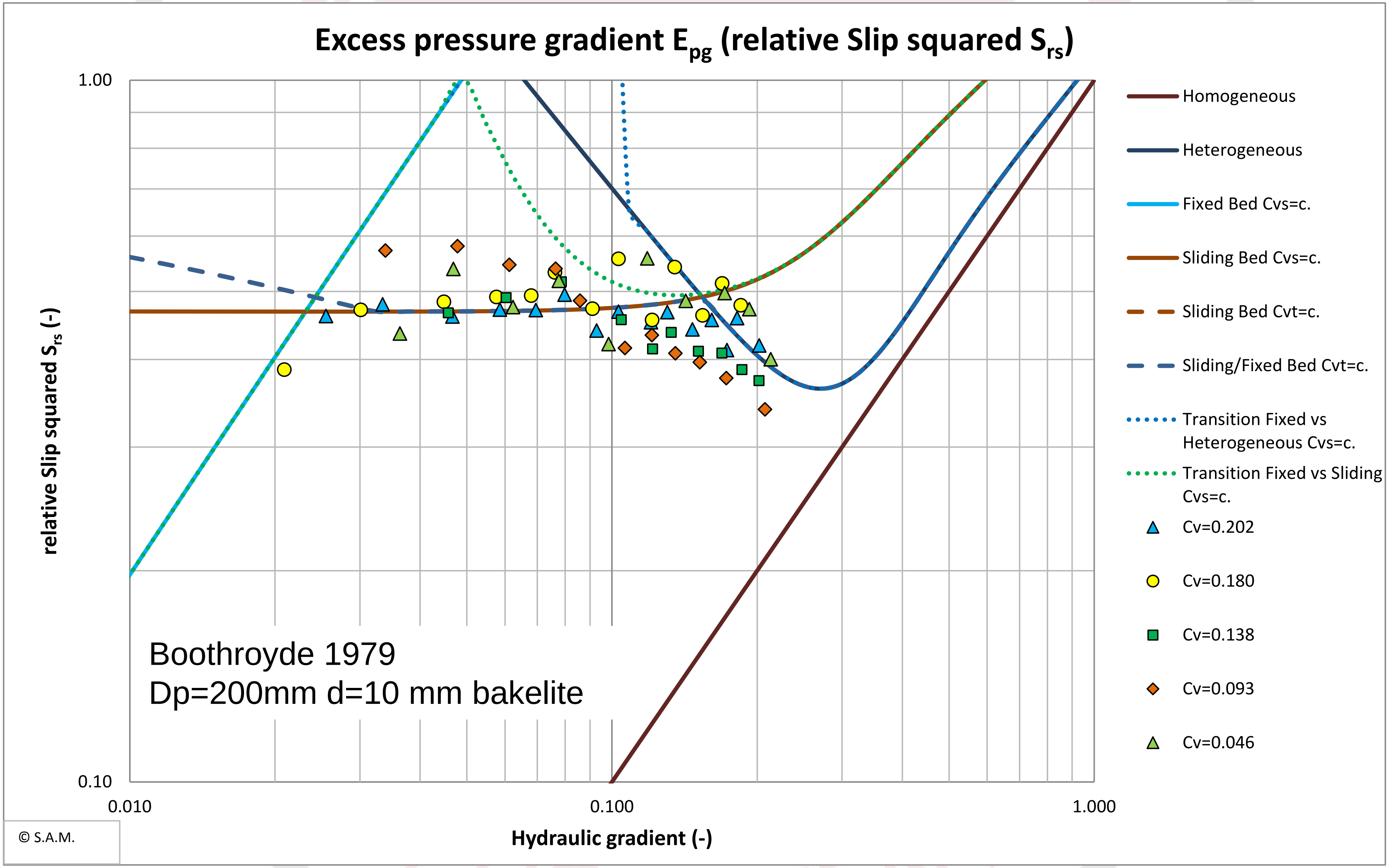


Validation



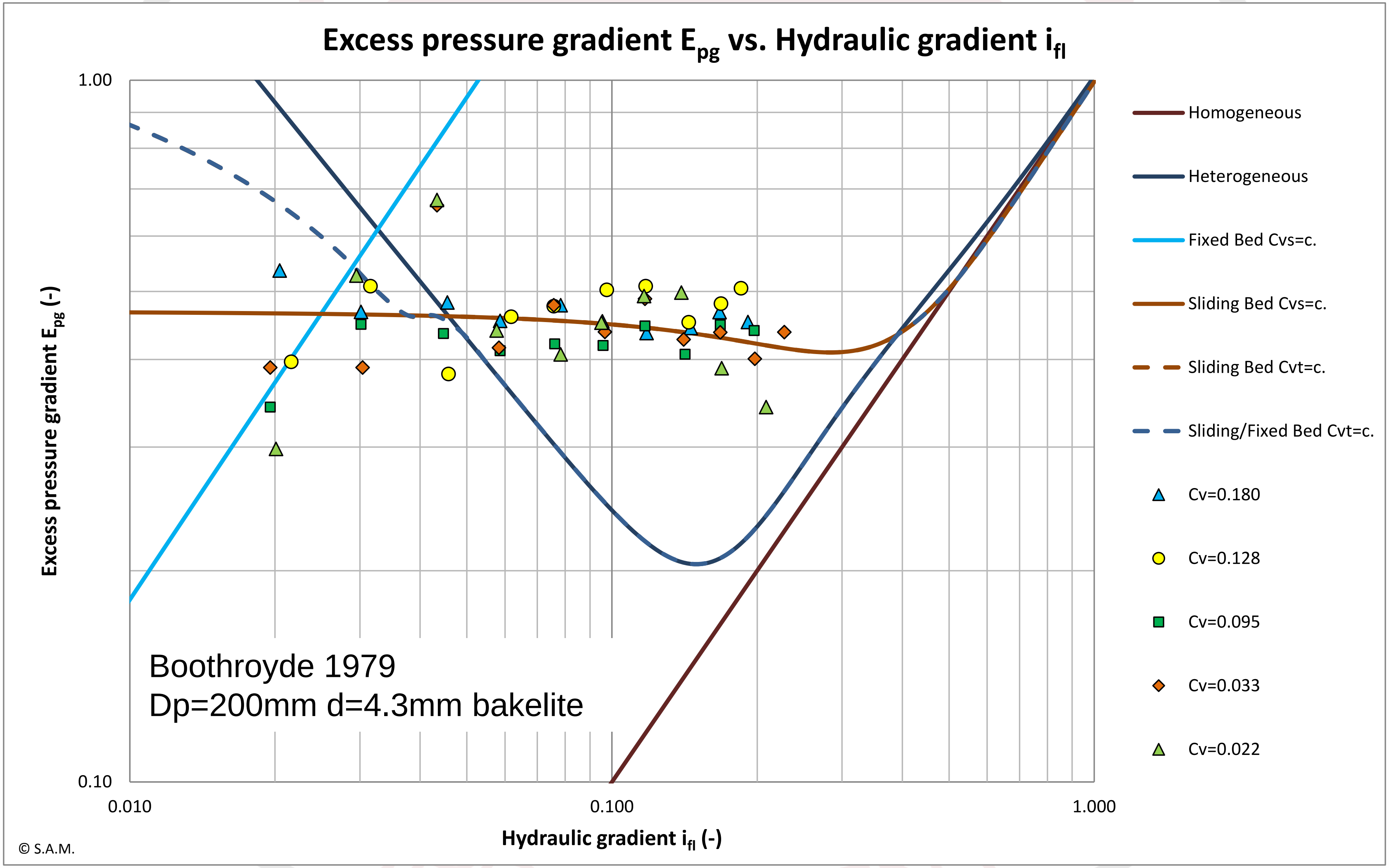
Excess pressure gradient E_{pg} vs. Hydraulic gradient i_{fl}
1.00
0.10
0.01
0.001
0.010
0.100
1.000
Excess pressure gradient E_{pg} (-)
Hydraulic gradient i_{fl} (-)
Homogeneous
Heterogeneous
Fixed Bed Cvs=c.
Sliding Bed Cvs=c.
Sliding Bed Cvt=c.
Sliding/Fixed Bed Cvt=c.
Cv=0.300
Cv=0.250
Cv=0.200
Cv=0.150
Cv=0.100
Cv=0.050
Wiedenroth 1967
Dp=300mm d=2.2 mm
© S.A.M.

Validation



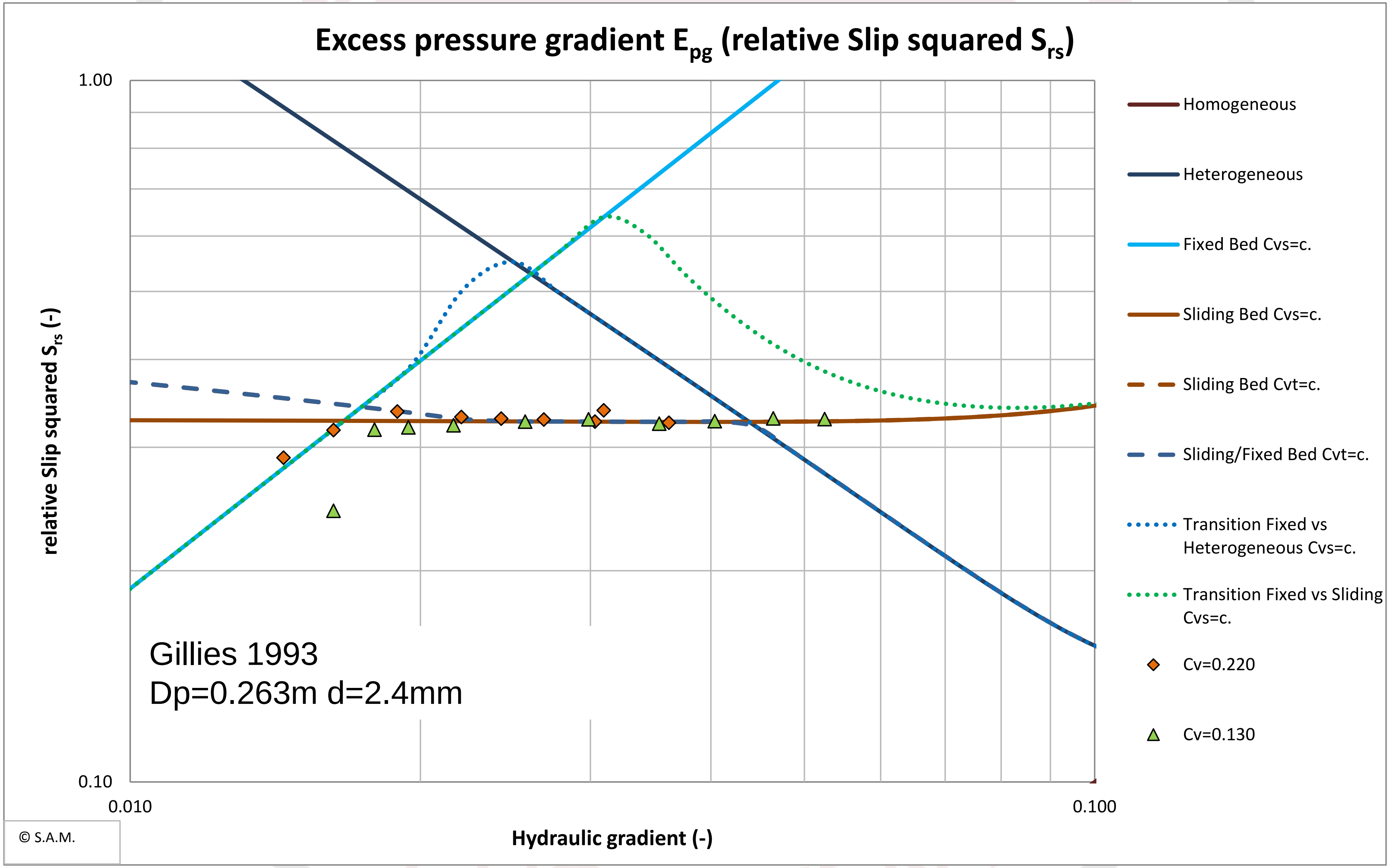
Excess pressure gradient E_{pg} (relative Slip squared S_{rs})
1.00
0.10
0.010
0.100
1.000
relative Slip squared S_{rs} (-)
Hydraulic gradient (-)
Boothroyde 1979
Dp=200mm d=10 mm bakelite
Homogeneous
Heterogeneous
Fixed Bed Cvs=c.
Sliding Bed Cvs=c.
Sliding Bed Cvt=c.
Sliding/Fixed Bed Cvt=c.
Transition Fixed vs Heterogeneous Cvs=c.
Transition Fixed vs Sliding Cvs=c.
Cv=0.202
Cv=0.180
Cv=0.138
Cv=0.093
Cv=0.046

Validation



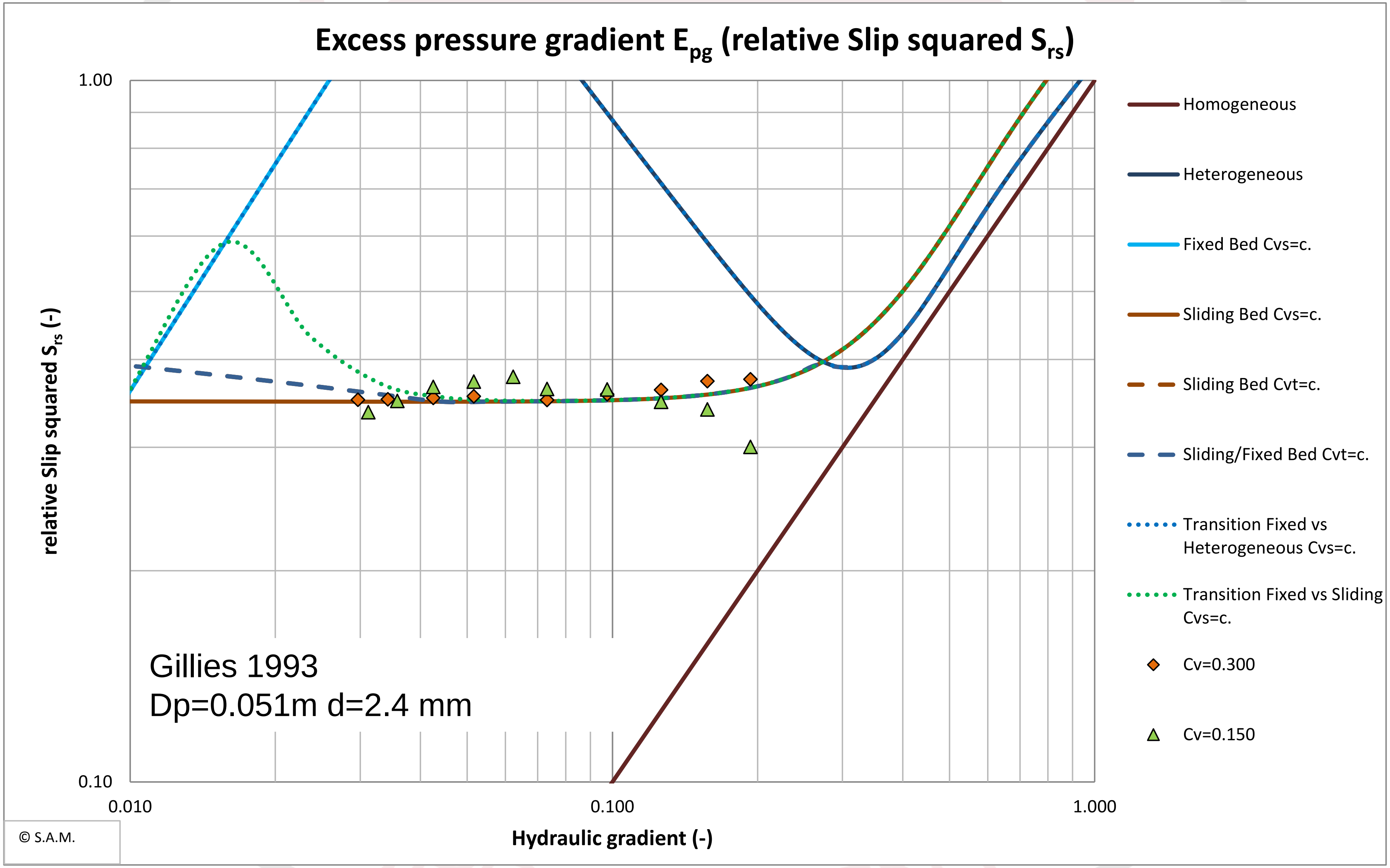
Excess pressure gradient E_{pg} vs. Hydraulic gradient i_{fl}
1.00
0.10
0.010
0.100
1.000
Excess pressure gradient E_{pg} (-)
Hydraulic gradient i_{fl} (-)
Homogeneous
Heterogeneous
Fixed Bed Cvs=c.
Sliding Bed Cvs=c.
Sliding Bed Cvt=c.
Sliding/Fixed Bed Cvt=c.
Cv=0.180
Cv=0.128
Cv=0.095
Cv=0.033
Cv=0.022
Boothroyde 1979
Dp=200mm d=4.3mm bakelite
© S.A.M.

Validation



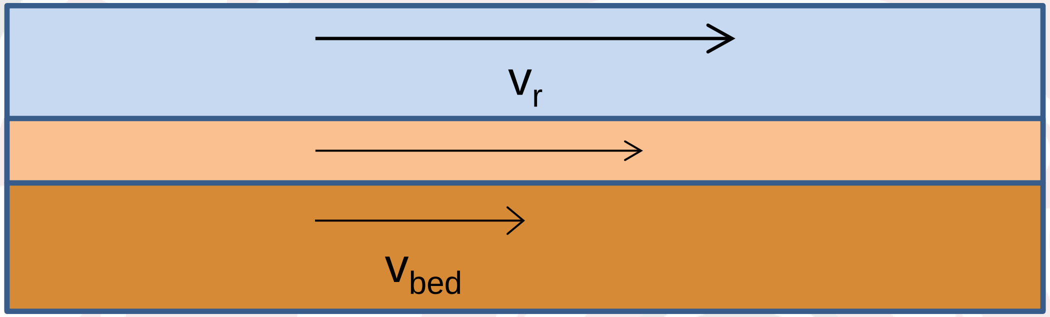
Excess pressure gradient E_{pg} (relative Slip squared S_{rs})
1.00
0.10
0.010
0.100
relative Slip squared S_{rs} (-)
Hydraulic gradient (-)
Gillies 1993
Dp=0.263m d=2.4mm
Homogeneous
Heterogeneous
Fixed Bed Cvs=c.
Sliding Bed Cvs=c.
Sliding Bed Cvt=c.
Sliding/Fixed Bed Cvt=c.
Transition Fixed vs Heterogeneous Cvs=c.
Transition Fixed vs Sliding Cvs=c.
Cv=0.220
Cv=0.130

Validation



Excess pressure gradient Epg (relative Slip squared Srs)
1.00
0.10
0.010
0.100
1.000
relative Slip squared Srs (-)
Hydraulic gradient (-)
Gillies 1993
Dp=0.051m d=2.4 mm
Homogeneous
Heterogeneous
Fixed Bed Cvs=c.
Sliding Bed Cvs=c.
Sliding Bed Cvt=c.
Sliding/Fixed Bed Cvt=c.
Transition Fixed vs Heterogeneous Cvs=c.
Transition Fixed vs Sliding Cvs=c.
Cv=0.300
Cv=0.150
© S.A.M.

Sheet Flow



$v_r \cdot A_r$

$v_{sheet} \cdot A_{sheet}$

$v_{bed} \cdot A_{bed}$

$$Total\ Flow = v_r \cdot A_r + v_{sheet} \cdot A_{sheet} + v_{bed} \cdot A_{bed}$$

$$v_{ls} = \frac{v_r \cdot A_r + v_{sheet} \cdot A_{sheet} + v_{bed} \cdot A_{bed}}{A_r + A_{sheet} + A_{bed}}$$

Stability Considerations

The Wilson and Normal Force approaches are equivalent up to ½ of the full pipe

Operation with a larger bed is generally to be avoided as it is very unstable and may lead to pipeline plugs

Conclusions – Regimes and Results

- 3 models for the friction between the bed and pipewall are presented.
- The Wilson and submerged weight approaches can be thought of as maximum and minimum conditions, with one assuming horizontal stress around the entire perimeter, and the other neglecting horizontal stress altogether.
- However, the experiments do not show a dependency of the apparent sliding friction coefficient with the volumetric concentration.
- Using a sliding friction factor between 0.35 and 0.45 seems realistic matching the experiments.

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Questions?